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INCORPORATION OF FORECASTED SEASONAL RUNOFF VOLUMES INTO RESERVOIR MANAGEMENT

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Water Resources Series
Technical Report No. 58
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by

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ABSTRACT

The literature for the broad subject area of seasonal snowmelt runoff volume forecasting was examined to determine what methods of forecasting show the most promise for providing data for improving reservoir operations. The forecasting approaches can be classified as regression methods, pseudo water balance methods, and water balance methods; different approaches have different data demands and different forecast accuracies. While forecasts of runoff volumes in a particular time interval (e.g., one month) may not be very accurate, the total seasonal forecast is relatively accurate. Consequently the major effort of this report focused on disaggregating predicted total seasonal runoff volumes to yield conditional probability distributions of flow in each month of the total forecast period. These conditional distributions were used as principal input data to a reservoir operation model.

Forecasts of total snowmelt seasonal runoff volumes were used to determine operating rules for hypothetical reservoirs to determine the relative advantage of using forecasted flow over the unconditional flow state. Comparisons were made using chance constrained linear programming (CCLP) formulations for the operation of a single multiple purpose reservoir (flood mitigation, water supply, and hydropower generation) with and without snowmelt runoff forecasts. Two reservoir sizes, one approximately 20% and the other 100% of the mean annual flow volume of a river, were examined. Flow data from the Cedar River, Washington, were used to reflect runoff patterns typical of much of the Pacific Northwest region.

Conditional streamflow distributions for months March-August were developed for given total March-August runoff volume amounts. These conditional distributions were used to develop Linear Decision Rules (LDR's) for the release from the reservoir. LDR's were also developed for the maximum ignorance state. Economic benefits were computed for maximum ignorance operating rules (Type A), Type A operating rules with conditional inflows (Type B) and LDR's and benefits based on the conditional flows (Type C). Generally, under Type C operation there was less uncertainty in the cumulative distributions of water supply and reservoir freeboard (surrogate for flood control) than when Type A operation was used. Explicit inclusion of the total seasonal runoff forecast into reservoir operation reduces physical operating uncertainty, as well as generates larger benefits than are determined by operating under circumstances of maximum ignorance, i.e., when only unconditional streamflow distributions are available.

Keywords: Snowmelt; Stochastic Hydrology; Optimization; Reservoir Operation

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CHAPTER 1: INTRODUCTION

This report examines some potential advantages of using total melt season runoff forecasts explicitly in reservoir operations. Operators of reservoirs that receive much of their supply from snowmelt have long recognized the importance of accurate estimates of meltable snow pack in tributary catchments. Numerous techniques exist for estimating the melt season runoff volume and these estimates are used with varying degrees of formalism depending on the size and scope of the reservoir or reservoir system. Most forecasts are still incorporated into empirically derived operating procedures which reflect the accumulated skills, experience, and judgment of the system operators. The major purpose of the current work was to investigate various forecasting schemes and to determine how a reliable forecast could be incorporated formally into reservoir operations. The formalization of system operations was explored for a single facility; the techniques used were developed with the clear understanding that they must also be practicable for multiple facilities.

Methods for forecasting snowmelt runoff volumes are investigated and discussed in Chapter 2. Two particular types of forecasting methods, those using deterministic precipitation-runoff models, and those using a storage balance accounting methods are examined in detail in a second report (Lettenmaier and Waddle, 1978).

The most reliable forecasts appear to be those that estimate total melt season runoff volume rather than, say , monthly flow volumes during the melt season. The covariance structure between monthly flows and the total snowmelt runoff volume (typically March to August in the Northwest of the U.S.) provides an opportunity to disaggregate total season forecast runoff to conditional flow distributions for the months included in the melt period. Monthly time increments are not sacred; they are convenient and suitable, however, for many systems.

Chapter 3 gives details of a procedure which was developed for disaggregating the total melt season forecast. Conditional distributions obtained via this approach incorporate monthly marginal distributions which are represented by three parameter log normal (3PLN) distributions. A principal components technique is used to ensure that relevant conditional marginal distributions are obtained.

There have been numerous efforts to determine the worth of a forecast. The greatest difficulty in such efforts is to determine the baseline for comparisons. Clyde (1951) and Nelson (1969) have estimated benefits for irrigation when forecasts are used. Hamon (1972) investigated the benefits to power generation and irrigated agriculture from improved forecasts in the Columbia River System. Fiering (1969) provided a formalism for using forecasts of varying reliability to determine system operating rules while he gave particular attention to the importance of explicit incorporation of economic benefits. Few generalizations could be made. While his approach might be suitable for evaluating the worth of a forecast, we elected to follow another broad path. Rather than attempt a large-scale evaluation of the type previously done, we decided to examine a hypothetical single multiple-purpose reservoir and compare expected benefits with and without a forecast. This approach necessitated selection of operating rules. Rules were developed using the Linear Decision Rule (LDR) approach, which was introduced to the water resources field by ReVelle et al. (1969). The approach used draws heavily upon developments of the LDR method may by Houck (1975).

The worth of a forecast issue is explored in Chapter 4. The comparisons indicate where less than average expected benefits are obtained as well as circumstances where greater financial return results. The importance of the approach is that contracts for water supply and power might be renegotiated because the forecast is available sufficiently early in the season to permit alternative purchases and sales to be made. This would perhaps be one of the greatest benefits of this type of forecast; different types of contracts for sale and purchase of power (and in some cases, water) than those currently in use would need to be explored.

Conclusions from this work and suggestions for future work are also given in Chapter 4. A listing of the computer code used to obtain the disaggregated conditional flow distributions is included in Appendix 1. Some of the subroutines make calls to the IMSL routines available at the University of Washington Academic Computer Center. Equivalent code may be substituted; many computer centers have access to IMSL routines, however. Details of IMSL code can be obtained from International Mathematical and Statistical Libraries, Inc., 7500 Bellaire Blvd., Sixth Floor, GNB Bldg., Houston, Texas 77036. Computer code for the chance constrained linear program (CCLP) operation model is not included because it is LP software dependent.

CHAPTER 2: METHODS FOR FORECASTING SEASONAL SNOWMELT RUNOFF VOLUMES

2.1 Introduction

Indirect evidence of the need for improved forecasts of snowmelt runoff volumes is provided by the increase in technical publications on this subject in recent years. For example, the Proceedings on the Role of Snow and Ice in Hydrology (1972) contains 22 papers that directly address the forecast problem. The top priority of forecasters is to improve the accuracy of seasonal volume forecasts, while at the same time working towards the goal of forecasting the complete hydrograph on as short a time increment as six hours to one day. Seasonal volume forecasts may be obtained by a single time period (total runoff season) analysis or by summing forecasts of shorter time periods. While forecasts of individual days may be greatly in error, the aggregation of these forecasts may yield quite satisfactory total seasonal forecasts, particularly if a moisture budget type of model is used.

The reason for the search for more accurate forecasts is, of course, improved management of water resources. Planning for agriculture, municipal water supply, energy generation, recreation, navigation, flood, and pollution control, all benefit from accurate forecasts. Actual relationships between forecast accuracy and monetary benefits are not well defined; attempts to relate the value of forecasts in terms of dollars have been made, however.

Clyde and Houston (1951) estimated that an additional $3.50 per acre (1950 dollars) was realized on farms that included, indeed depended, on the water supply forecast as a critical element in crop planning and water management. Nelson (1969) calculated that an additional $10 per acre (1968 dollar) can be realized by using water supply forecasts for planning purposes. He further estimated that the total benefit realized, considering the 13 million irrigated acres in the west (excluding Bureau of Reclamation projects), is between 32.5 million and 65 million dollars. He claims that these figures are conservative estimates of the total economic value of water supply forecasts to western agriculture. Hamon (1972) calculated the effects of an improvement in the January 1 flow forecast for the Columbia River at The Dalles, Oregon, on power production and irrigated agriculture. His calculations produced an additional return of 6.2 million dollars (1971 dollars) annually for a 1 percent improvement in the forecast.

More recently, Schramm, et al. (1974), using a probabilistic linear programming (LP) model which maximized the expected income of plantings associated with various forecasts, estimated gross benefits ranging from a few cents up to $6.00 per acre, depending on the economic values in the use area. The second portion of this same study focused on the effects of more accurate forecasts on the efficiency of multi-purpose reservoir operation. Results indicate that, in general, improved water supply forecasts will result in more efficient reservoir operations and larger dollar and public good benefits. (A general probabilistic LP which uses stochastically dependent seasonal flow volumes, conditioned on a seasonal forecast, is developed and discussed here in Chapter 4.)

2.2 Seasonal Water Supply Forecasts

The oldest and most widely used approach to seasonal water supply forecasting in areas of seasonal snow cover is embodied in a linear model of the form

$$Y = a + B_1X_1 + B_2X_2 \; . \; . \; . \; .B_nX_n \tag{2.1}$$

where Y is the runoff volume for a specific forecast period, X_1 through X_n are runoff index variables, a is a fitting coefficient, and B_1 through B_n are weighting factors for each runoff index variable. Basically, the procedure involves the correlation of the historical runoff record with the index variables. The most significant of the index variables is snow water equivalent of the snowpack, as measured at one or several snow courses. In many western U.S. river basins, the snow water equivalent parameter explains between 60 and 90 percent of the variance in runoff volume during the snowmelt season. This type of model can suffer from problems of colinearity if too many runoff index variables are used and they are not mathematically orthogonal.

The Soil Conservation Service National Engineering Handbook (1970), identifies the index variables in use, or those of potential use, as: snow water equivalent, antecedent streamflow, baseflow, soil moisture, precipitation (fall, winter, and spring), temperature, wind, solar radiation, and relative humidity. This is also in general agreement with a similar list published by the U.S. Army Corps of Engineers in Snow Hydrology (1956). In both publications, the suggested method of calculating weights to apply to the index variables and determining the fitting coefficient is the use of multiple linear regression techniques, with some emphasis placed on graphical correlation. In practice, weights for the index variables were often determined subjectively or through trial and error methods; the current trend is to use statistical methods (Coulson, 1970; Schermerhorn and Barton, 1968).

Marsden and Davis (1968) recognized the previous incorrect uses of multiple regression and used a principal components approach to orthogonalize their multiple regression to calculate relevant weights for principal components based on the usual runoff index variables. They demonstrated improved forecast accuracy for the Yakima Basin, Washington. (A better approach would be to perform a stepwise regression on the runoff index variables and then use those selected variables in a principal component regression model.) Zuzel et al. (1975) and Zuzel and Ondrechen (1975) used a pattern search optimization technique to produce more accurate forecasts than the methods then currently in use for the Boise River in Idaho. In this method, snow water equivalent and/or precipitation weights are constrained so that the forecast equation is forced to conform to a general water balance equation and snow water equivalent at the snow course is expressed as a depth-area function. This allows average snow water equivalent to be related directly to runoff volume for any forecast period.

The question of which variables to use in any forecast model has been a subject of much discussion, but in the final analysis, is probably dependent on the topographic, climatic, geologic, soils, and vegetative characteristics of each basin. The most commonly employed runoff index variables are snow water equivalent, measured at a snow course site, and precipitation, measured at a snow course site or at a valley station. These two parameters are common to nearly all seasonal streamflow models.

Although not a recent innovation, the use of aerial photography to estimate snow cover has assumed more importance since the advent of satellite photography. Rango and Salomonson (1975), and Rango et al. (1977) used LANDSAT-I imagery to determine snow cover over the upper Indus Basin, Pakistan, and the Wind River Mountains of Wyoming. Using these data and seasonal streamflow, regression equations were developed which produced remarkably high correlation coefficients between snow cover and seasonal runoff. (For the Indus River, 1969-1973, $r^2 = 0.82$, and for the Kabul River, 1967-1973, $r^2 = 0.89$).

An evaluation of aerial photography by image-analyzing computer has been explored by Martinec (1972). Leaf (1969) developed a depletion-runoff relationship, based on snow cover, calculated from aerial photographs. The major objections to either satellite or aircraft photography are the excessive turn-around time for photo processing the large number of cloud-covered days and the inability of photointerpreters and image analyzing computers to definitely distinguish between clouds and snow.

Fletcher (1966) discussed the importance of soil moisture data for inclusion among the index variables. However, the difficulties of obtaining routine soil moisture measurements have never really been overcome. Resistance and capacitance measurements have a tendency to change calibration with time and neutron moderation equipment is simply too expensive and delicate for routine use. As a result of these measurement difficulties, soil moisture data are not commonly used in forecasting.

A procedure for correcting snow water equivalent measurements for evapo-sublimation was introduced by Peak (1969). Using wind, solar radiation, and temperature data, he succeeded in reducing the forecast error for several Wyoming watersheds. However, during the same year, Doty and Johnston (1969), in a research study conducted in Utah, reported that the amount of winter evapo-sublimation was compensated for by the amount of condensation and that the net effect of the two processes was essentially zero. Also, while temperature measurements are collected routinely at most remote sites, wind and solar radiation are not and valley stations must be used for obtaining these parameters. consequently, forecast methods, using these index variables, are not in common use. It would also appear that while solar radiation provides the heat source necessary for evapo-sublimation and wind is a good indicator of the degree of turbulent transfer occurring, the most important parameter, the direction and magnitude of the vapor pressure gradient, has been overlooked.

Antecedent streamflow and/or baseflow are incorporated into forecast models much more frequently, because streamflow does effectively index the soil moisture conditions on some watersheds (Snow Hydrology, 1956; George, 1970; Warnick and Brockway, 1974). Another index method is the coastal stream index. In this method, the winter runoff of low elevation streams is related to the spring snowmelt runoff. Precipitation at valley stations is sometimes included as an additional index variable. The method is reported to be as accurate as those that use snow water equivalent and precipitation data, and is fully described in Snow Hydrology (1956). A recent modification of this method and application to some Western Washington watersheds is described by Tangborn and Rasmussen (1976). The coastal index procedure is also confined to use in areas that have a common moisture source, such as the coastal basins of Washington and Oregon.

Index forecasting methods have probably reached the point where large increases in accuracy are not possible; slight improvements in accuracy may be obtained, however. Wilson (1966) suggests that more accurate forecasts can be obtained by using point samples of snow water equivalent, which are highly correlated with runoff, rather than snow course average snow water equivalent. It has also been suggested that accuracy can be improved by adjusting the calculated forecast on the basis of "feel," since the forecaster is in possession of certain facts, such as "no snow on south slopes" or "snow line elevation higher than other years" (Malsor, 1967). (This kind of adjustment is also suggested in Section 22 of the Soil Conservation Service National Engineering Handbook.) This form of art is user dependent and is seldom transferable to other locations.

The success or failure of computer-based watershed models is not readily apparent from the literature. Pearson (1974) used the Streamflow Synthesis and Reservoir Regulation (SSARR) model, developed by the U.S. Army Corps of Engineers, to simulate inflows to Hungry Horse reservoir in Montana.

He states that the model can be used in volumetric forecasting, based on the agreement between cumulative values of simulated versus actual inflows over the forecast period. A computer-based water budget model, reported by Quick and Pipes (1972), has been successfully used in forecasting both daily and seasonal runoff volumes. In this model, long-term forecasts require a knowledge of antecedent conditions, as well as snow water equivalent data. A desirable feature of this model is the ability to input weather data from previous years and create various scenarios so that upper and lower bounds can be attached to the snowmelt runoff forecast.

Numerous references to snowmelt runoff simulation can be found in the literature (Riley, et al., 1971; Anderson, 1973), but reports on the operational use of these models are almost completely lacking. Simulation models, while used extensively in California (principally through the efforts of R. Burnash and colleagues at the National Weather Service River Forecast Center, Sacramento, California) have not displaced the older index techniques that are entrenched in other forecasting agencies.

2.3 Short-Term Forecasting

Short-term rorecasts are considered here to be for daily, weekly, and monthly snowmelt runoff volumes. The most important issue is the accurate estimation of temporal and spatial snow pack properties and snowmelt volumes. Use of a good precipitation-runoff continuous simulation model with these estimates can provide accurate streamflow forecasts. Usually a model of the form of the basic Stanford Watershed Model (Crawford and Linsley, 1966) or the Sacramento Model which is used by the National Weather Service (Burnash et al., 1973) is needed for this purpose. The most difficult part of the forecasting effort is to estimate water supply delivered from precipitation and melt to soil storages and the channel system. Lack of data on direct causes of snowmelt has necessitated use of surrogate measures.

In established procedures a surrogate index is used to relate snowmelt directly to streamflow. More recent efforts, however, separate snowmelt dynamics from soil-water-stream dynamics. The established approaches usually relate air temperature at an index site to daily streamflow and, from this relationship, predict daily streamflow volumes one to several days in advance (Zimmerman, 1972). The objection to this procedure is that, while temperature is the best single index to snowmelt and thus runoff, it largely ignores the energy inputs required for the melting of the snow pack.

Zuzel and Cox (1975) found that in a 100 percent snow cover situation, more than 60 percent of the snowmelt is the direct result of net allwave radiation. These data support findings of Dewalle and Meiman (1971), who obtained similar values although their data were collected over a 2-day period only.

As the melt season progresses, and more and more soil and vegetative surfaces lose their snow cover, advected sensible heat contribures more to the melt process, and may become the dominant heat source (Gray and O'Neill, 1973; Cox and Zuzel, 1976). The relationship between the various heat sources and snowmelt is, therefore, a changing one and snowmelt can be calculated precisely only by approximating these heat sources for varying situations. Anderson (1976), in comparing the energy balance approach with the temperature index method, concluded that the energy balance method does enable one to calculate snowmelt more precisely. The strongest objection to using the energy balance method is the difficulty of measuring the required variables at remote sites. The objection is a valid one, although more rugged and reliable instrumentation is becoming available. Alternatively, energy balance parameters can be approximated, at least grossly, from the more readily available data, obtained from valley stations, and then corrected for elevation and exposure.

Snowmelt can also be measured at several index sites on a watershed by using snow pillows, isotopic snow gages, or snowmelt lysimeters, in conjunction with a telemetry system. Pysklywec, et al. (1968) and Haupt (1969) suggested that melt plots and/or lysimeters are efficient methods for measuring point snowmelt at index sites. However, snow pillow and snowmelt collectors can be unreliable in measuring diurnal changes in snow water equivalent (Cox, 1971). Also operational isotopic snow gages are insensitive to small changes in snow water equivalents.

Considering the above arguments, the potential advantage of deterministic watershed models for short-term forecasting becomes apparent. Varied climatological sequences can be used as input, along with known snow water equivalent and other antecedent variables. From this, short-term forecasts can be stated in probabilistic rather than deterministic terms. The routine use of such models will probably not come about until more reliable methods of computing or measuring snowmelt are perfected.

An important activity is to update models with directly measurable data as the data become available. It is well known that the accuracy of representation of the snow pack is essential for accurate forecasting. Carroll (1978) reported the current approach used by the National Weather Service to update their deterministic model to have the estimated snow pack agree with snow course data as the data become available. This is a precursor of future efforts in state-estimation modeling representations of snowmelt runoff. While such short term modeling activities are important for many aspects of water resources management, their major significance in the work reported here is for their abaility to yield accurate aggregate volumes typically over six months. Directions in water supply forecasting are well summarized by Tarble and Burnash (1971).

2.4 Summary

The forecast models discussed in Sections 2.2 and 2.3 use basic inputs of snow water equivalent, precipitation, and temperature. They can be classified as regression methods, pseudo water balance methods (pattern search optimization and coastal index) and water balance methods. Data requirements necessary for estimates of streamflow volume increase as one proceeds from the simple linear regression to the complete watershed model. The most stringent data requirements are associated with the watershed model since all hydrologic and meteorologic inputs and processes must be measured or estimated.

The cost of data acquisition must be considered when choosing a model for a particular basin, although this might be minimal in comparison to the benefits associated with an improved forecast. If, for example, the same forecast accuracy can be obtained using precipitation data as opposed to snow course data, it would be well to use only precipitation data due to the large difference in the acquisition costs. (For example, on the Boise River in Southwest Idaho, five snow courses are measured every two weeks by helicopter. Data collection costs are approximately $10,000 for a snow season of six months. This figure does not include data processing costs, or site maintenance (Wilson, 1977).

On the other hand, Tangborn and Rasmussen (1976) using only precipitation and runoff data have demonstrated some improvement in forecasting the Baker, Skagit, and Stehekin river flows in the Northern Cascades. The basic Tangborn approach does not necessarily remove the need for snowcourse data. For example, Lettenmaier (1978), using stepwise regression and principal components techniques, found some situations where snow course data were more useful than precipitation data in the basic Tangborn type of model.

Despite the limitations of forecasting, seasonal runoff volumes can be estimated (with varying accuracy) by one of the methods discussed here. This forecast can be used, together with its uncertainty, in many water resource management applications. An approach for demonstrating the worth of a forecast is given in Chapter 4. Other broader scale approaches have been used, e.g., to show the value of snow survey data to water supply forecasting (Elliot, 1977). The remainder of this report emphasizes how forecasts, regardless of how they are made, can be incorporated explicitly into reservoir operating rules. We have not taken operating constraints typically imposed under Federal Power Licensing agreements as binding. Rather, we have examined what might be achieved given the tools that are now available.

CHAPTER 3: DISAGGREGATION OF FORECASTED TOTAL SEASONAL RUNOFF VOLUMES

3.1 Introduction

A versatile family of time series models of the ARIMA type proposed by Box and Jenkins (1970) can be used for streamflow forecasts; applications of this approach have been discussed by McKerchar and Delleur (1974). These techniques are limited, however, to short-term applications, typically, of the order of several months, because no advantage is achieved when the forecasted conditional probability distribution of flow in a future time period coincides with the historical unconditional probability distribution for that period. Using an autoregressive model, Burges and Johnson (1973) have indicated that useful forecasts up to about four time periods ahead from the last observed period can be achieved for situations where interperiod correlations are greater than 0.5.

When a large fraction of spring and summer runoff results from release of water from a snow pack, forecasts for a longer time horizon can be made. While numerous methods for forecasting total seasonal runoff are available, most are of the multiple-regression type (see Chapter 2) and incorporate snow course information, soil moisture, and antecedent precipitation among other variables. While the forecasted quantity is not known with precision, considerable information results from the forecast. Rasmussen and Tangborn (1976), and Tangborn and Rasmussen (1976) among others, have developed some effective forecasting methods which are physically based. These forecasts can be used to obtain conditional flows. Disaggregation of a given forecasted seasonal volume into, typically, monthly flow volumes provides conditional streamflow information (different from the unconditional historical monthly flows) which can be used with appropriate reservoir operation algorithms.

The disaggregation scheme developed here takes advantage of the flow covariance structure. Skewed monthly marginal distributions are modeled using three parameter log normal distributions (3PLN's) (Aitchison and Brown, 1957; Fiering and Jackson, 1971; Burges et al., 1975) that are force-fitted to the data. The conditional distribution procedure is developed for a single site application where typically n conditional flow distributions are sought. Usually, $n \leq 6$ months, hence matrices have dimension $n \times n$ (6×6). Extension to k sites would require use of matrices smaller than ($6k \times 6k$).

3.2 Conditional First Three Moments

Let Q be a random variable representing forecasted total melt season runoff volume. (Here this is the cumulative volume of runoff forecasted for $n = 6$ months from March to August.) Let Q_0 be a particular value of Q, and let R_t be a random variable describing runoff volume in the t^{th} month of the forecast period. The first two conditional moments for flow in time period t given $Q = Q_0$ are

$$m_t = E(R_t|Q_0) = \mu_t + \rho_{Qt} \frac{\sigma_{R_t}}{\sigma_Q} (Q_0 - \mu_Q) \qquad \forall\ t \tag{3.1}$$

and

$$V_{tj} = \mathrm{Cov}(R_t, R_j|Q_0) = \sigma_{R_t} \sigma_{R_j} (\rho_{tj} - \rho_{Qt}\rho_{Qj}) \qquad \forall\ t,j \tag{3.2}$$

where $E(R_t|Q_0)$ is the conditional mean of flow R_t; $\mathrm{Cov}(R_t,R_j|Q_0)$ is the conditional covariance between R_t and R_j; μ_t and μ_Q are the unconditional means of R_t and Q, respectively; σ_{R_t} and σ_Q are the unconditional standard deviations of R_t and Q, respectively; ρ_{Qt} is the unconditional correlation coefficient between Q and R_t; and ρ_{tj} is the unconditional correlation coefficient between R_t and R_j.

Equation (3.2), with $t=j$ yields the conditional variance of R_t:

$$V_t^2 = \mathrm{Var}(R_t | Q_0) = \sigma_{R_t}^2 (1 - \rho_{Qt}^2) \qquad \forall\ t \tag{3.3}$$

Equation (3.3) is a well-known linear regression result for two variables; no account has been taken of the covariance information between the several R_t. Equations (3.2) and (3.3) show that the conditional covariance is independent of Q_0 and that the conditional variances are identical with the second moments of the independent random error terms in linear regression models.

The conditional skew coefficient is the normalized third moment of these error terms. For homoscedastic (constant variance) error terms, the conditional skew coefficient of R_t, γ_t, becomes

$$\gamma_t = \frac{G_{R_t} - \rho_{Qt}^3 G_Q}{(1 - \rho_{Qt}^2)^{3/2}} \qquad \forall\ t \tag{3.4}$$

where G_{R_t} and G_Q are the unconditional skew coefficients of R_t and Q, respectively. Equation (3.4) shows that γ_t is zero when R_t and Q are symmetrically distributed. In hydrologic modeling this results when R_t and Q are normally distributed. γ_t becomes numerically larger than G_{R_t} in compensating for reduction in the forecast error (i.e., smaller conditional variance). This is evident because of the form of the denominator in (3.4). γ_t does not necessarily have the same sign as G_{R_t} (which is usually positive for streamflow data). Equation 3.4 has been widely used in synthetic streamflow models to model skewed marginal distributions [see, e.g., Fiering and Jackson (1971)].

3.3 Non-Normally Distributed Seasonal and Forecasted Flows

In equations (3.1) to (3.4) no assumptions were made about the probability density functions of R_t and Q. However, if quantitative probability statements of the forecast inflow in month t are to be made, appropriate distributions have to be used. Transformation of non-normally distributed forecasted flow inflow volumes, Z, having the first three conditional moments m_t, V_t^2, and γ_t, to normally distributed quantities, Y, is necessary when the conditional distribution of forecasted inflow in a particular forecast time period t is required. In most cases, streamflow data can be approximated by force fitting a 3PLN distribution to them. While the overall fit may not be satisfactory, it is usually possible to fit the portion (low or high flows) of critical concern quite well. This general distriubtion is used because it is possible to approximate Gaussian [see, e.g., Burges and Hoshi (1978)] as well as skewed distributions with one general transformation. This is most important when mixed distributions are encountered. The convenience of the 3PLN distribution lies in the fact that there exist theoretical relationships between statistical parameters in the untransformed and transformed domains.

3.3.1 γ_t Positive

Let Z_t and Y_t be the forecasted inflow (untransformed) and normally distributed (transformed) inflow in period t. Summary statistics m_t, V_{tj}, and γ_t for the conditional mean, covariance (months t and j) and positive skew, respectively, transform under the 3PLN relationships to

$$m_t = a_t + \exp(d_t + U_t^2/2) \qquad \forall\ t \tag{3.5}$$

$$V_t^2 = [\exp(U_t^2) - 1]\exp(2d_t + U_t^2) \qquad \forall\ t \tag{3.6}$$

$$\gamma_t = \frac{\exp(3U_t^2) - 3\exp(U_t^2) + 2}{[\exp(U_t^2) - 1]^{3/2}} \qquad \forall\ t \tag{3.7}$$

$$V_{tj} = [\exp(U_{tj}) - 1]\exp(d_t + d_j + U_t^2/2 + U_j^2/2) \tag{3.8}$$

where d_t, U_t^2, a_t, and U_{tj} are the conditional mean, variance, third parameter, and covariance (periods t and j) of the normally distributed quantities, Y_t.

The transformation relationship is

$$Y_t = \ln(Z_t - a_t) \qquad \forall\ t \tag{3.9}$$

3.3.2 γ_t Negative

The conditional covariance defined by (3.8) is determined when both forecasted flows, Z_t and Z_j, are 3PLN distributed with positive skewness coefficients. In situations where the conditional skewness γ_t is negative, the transformation

$$Y_t = \ln(a_t - Z_t) \tag{3.9a}$$

produces normally distributed quantities, Y_t.

The conditional mean relationships become:

$$m_t = a_t - \exp(d_t + U_t^2/2) \qquad \forall\ t \tag{3.5a}$$

The conditional variance (3.6) is independent of the sign of γ_t. When γ_t is negative $|\gamma_t|$ is used in (3.7) to compute U_t. When both γ_t and γ_j are negative [i.e., (3.9a) is used to effect transformation], (3.8) is used to compute U_{tj}. When γ_t and γ_j have different signs, $-V_{tj}$ is used in (3.8) to compute U_{tj}.

3.3.3 Computational Considerations

The conditional mean, variance, and covariance d_t, U_t^2, and U_{tj} in the transformed normal domain and the third parameter, a_t, can be determined using equations (3.5)-(3.8) for the given values of m_t, V_t^2, γ_t, and V_{tj}, depending on the specified value of $Q = Q_0$. As stated above, however, the conditional covariance and skewness are independent of Q_0. The only variables in (3.5) to (3.8) that are influenced by Q_0 are a_t and d_t.

If the above approach is not followed exactly it will not be possible to preserve the distribution-free conditional means and variances for each, t, defined by (3.1) and (3.2), because of the nonlinear transformations involved. Moreover, the conditional skew coefficients cannot be preserved unless this approach is followed.

3.4 Determination of Conditional Marginal Distributions

Sections 3.2 and 3.3 discussed relationships between moments, but not those between the Y_t. Transformations are effected to yield normally distributed quantities; the existence of the conditional covariance structure means that Y_t is drawn from a multivariate normal distribution. The remainder of this chapter describes how relevant conditional marginal distributions are obtained; the conditional covariance structure is important and must not be overlooked.

To incorporate the conditional covariance into probability statements of the forecasted inflows in period t, the following theorem (Feller, 1971) is introduced:

There exists an orthogonal matrix C with determinant 1 (rotation matrix) such that a column vector Y is decomposed into a mutually independent normal column vector G with transformation of

$$Y = C\, G \tag{3.10}$$

where $Y^T = [y_1\; y_2 \ldots\; y_t \ldots\; y_n]$ (3.10a)

$$G^T = [g_1\; g_2 \ldots\; g_t \ldots\; g_n] \tag{3.10b}$$

C is an $(n \times n)$ coefficient matrix; n is the number of months in the forecast time period. Let Y be distributed as jointly normal with a column vector of conditional means (d_t, $\forall\, t$) and a square matrix of conditional covariances (U_{tj}, $\forall\, t,j$), i.e.,

$$d_Y^T = [d_1\; d_2 \ldots d_t \ldots d_n] \tag{3.11}$$

$$U_Y = \begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1n} \\ U_{21} & U_{22} & \cdots & U_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & U_{tj} & \cdots & \\ U_{n1} & U_{n2} & \cdots & U_{nn} \end{bmatrix} \tag{3.12}$$

where d_Y is a mean vector and U_Y is a covariance matrix.

Taking expectations in (3.10) yields

$$d_Y = E(Y) = C\ E(G) = C\ d_G \tag{3.13}$$

where $E(\cdot)$ is the expectation operator and d_G is a column vector whose elements are the mean values given in (3.10b).

Subtracting (3.13) from (3.10) yields

$$Y - d_Y = C(G - d_G) \tag{3.14}$$

It is clear that both the vectors $(Y - d_Y)$ and $(G - d_G)$ have zero mean elements. Postmultiplying (3.14) by $(Y - d_Y)^T$ and taking expectations gives

$$\begin{aligned} U_Y &= E[(Y - d_Y)(y - d_Y)^T] \\ &= C\ E[(G - d_G)(G - d_G)^T]\ C^T = C\ D_G\ C^T \end{aligned} \tag{3.15}$$

where D_G is the $(n{\times}n)$ covariance matrix of G. From the above theorem it follows that D_G has to be a diagonal matrix whose components consist of the variances of variable g_t. Specifically,

$$D_G = \begin{bmatrix} \sigma^2_{g_1} & & & \\ & \sigma^2_{g_2} & & 0 \\ & & \ddots & \\ 0 & & & \sigma^2_{g_n} \end{bmatrix} \tag{3.16}$$

Using the relationship given by (3.16), (3.15) reduces to

$$U_Y = \left(C\, D_G^{1/2}\right)\left(C\, D_G^{1/2}\right)^T = C_* \, C_*^T \tag{3.15a}$$

where $D_G^{1/2}$ is a diagonal matrix whose components are the standard deviations of variable g_t, and

$$C_* = C\, D_G^{1/2} \tag{3.17}$$

Equation (3.15a) is frequently encountered in multivariate stochastic generation models in which the solution for C_* can be effected by the method of principal components. As a result, the elements of diagonal matrix, D_G, are given by the eigenvalues of the symmetric matrix, U_Y; the rotation matrix, C, is derived from a matrix whose columns are eigenvectors corresponding to each eigenvalue of U_Y. The rotation matrix, C, has the important property that

$$C^T = C^{-1} \tag{3.18}$$

and

$$c_t^T c_j = \begin{cases} 1 \ (t = j) & \forall\ t,j \\ 0 \ (t \neq j) & \forall\ t,j \end{cases} \tag{3.19}$$

where c_t is the t^{th} column vector of C.

Using (3.13) and (3.18), the mean vector of variable G is given by

$$d_G = C^T\, d_Y \tag{3.20}$$

Because the elements of the column vector, G, are mutually independent, the element, g_t, is distributed as univariate normal with mean d_{gt}, determined from (3.20), and variance σ_{gt}^2 (equivalent to an eigenvalue of U_Y). Physical inflow volumes corresponding to cumulative probability, p,$(0 \leq p \leq 1)$, (in the G domain) that flow is less than or equal to a specified flow can be easily computed from

$$(g_t)_p = d_{g_t} + t_p \, \sigma_{g_t} \qquad \forall\, t \tag{3.21}$$

where $(g_t)_p$ is the p^{th} percentile flow of g_t, and t_p is the standard normal variate corresponding to the probability level, p.

The p^{th} percentile of Y_t (in the Y domain) is determined by using the rotation matrix to be

$$(Y_t) = \sum_{j=1}^{n} c_{tj} \, (g_j)_p \qquad \forall\, t \tag{3.22}$$

where $(y_t)_p$ is the p^{th} percentile flow of y_t, and c_{tj} is the $(t,j)^{th}$ element of rotation matrix, C.

Finally, the 3PLN distributed forecast inflow volume corresponding to the cumulative probability, p, is inverse transformed to yield

$$(z_t)_p = a_t + \exp[(y_t)_p] \qquad \forall\, t$$

$$\gamma_t \geq 0 \tag{3.23}$$

and

$$(z_t)_p = a_t - \exp[(y_t)_p] \qquad \forall\, t$$

$$\gamma_t < 0 \tag{3.23a}$$

where $(z_t)_p$ is the p^{th} percentile flow in the untransformed domain.

The complete conditional distribution (given $Q = Q_0$) is obtained by substituting into (3.21)-(3.23) for several values of p. As shown in Figures 1 and 2 (Chapter 4), nine values of p sufficed to compute the distributions of interest.

A FORTRAN program suitable for computing conditional distributions is given in Appendix 1. This program was implemented on a CDC 6400 digital computer at the University of Washington Academic Computer Center. Machine dependent routine CLOCK was used for triggering the CDC uniform random number generator RANF. Normal random deviates were obtained from the uniform random numbers. Calls are made to IMSL supplied subroutines. Users at other installations would need to substitute equivalent routines if IMSL routines are not available. The code is listed here for other interested persons; too few comments are included to make the program fully understandable to the general reader.

3.5 Summary

Conditional distributions of streamflow volume for a sub-interval (month) within an overall forecast interval (snowmelt runoff season) can be obtained by following the mathematically defined procedures in this chapter. It is important to reemphasize here that the information contained in the conditional variance structure must be included when computing conditional marginal flow distributions. The nonlinear transformations introduced by approximating untransformed flow data with 3PLN distributions necessitate use of the exact moment relationships used here.

CHAPTER 4: USE OF FLOW FORECAST IN RESERVOIR MANAGEMENT

4.1 Introduction

There are many well-known problems associated with operating single multiple purpose storage reservoirs. In the past 10 years many investigators have developed different methods for determining operating rules. Of all the attempts made so far, it appears from both practical considerations (concerning how well a given system may be represented) and computability that linear programming formulations have much to offer. In particular, Linear Programming (LP) formulations that use Linear Decision Rules (LDR's) offer many advantages.

While much as been done with LDR formulations, they have suffered from the limited ways that stochasticity of streamflow has been incorporated; unconditional marginal flow distributions have been used to determine chance constraints. A further limitation stems from the fact that rules are developed for statistically average future scenarios rather than forecasted flow scenarios. Long-term forecasting (many years) is currently not feasible. It is feasible, however, to make seasonal forecasts, e.g., forecasts of total snowmelt (spring and summer) runoff volumes.

The principal emphasis here is to show how the operation of a single multiple purpose reservoir can be formulated as a chance constrained LP with LDR's developed for conditionally distributed monthly flows which are based upon a specific seasonal runoff forecast. The method is based upon work by Houck (1975) which permits computation of both reservoir release rules as well as the expected annual benefit which results from the forecast. The method extends earlier LDR formulations in that conditional flow distributions are explicitly used. Flow data from the Cedar River, Washington, were used to reflect flow statistics representative of much of the Northwest region of the United States.

Two hypothetical reservoirs were examined, one had capacity equal to the mean annual flow volume, μ, and the other had a capacity of $0.2\ \mu$. Optimal facility operation was the objective, hence all capital costs were assumed to be sunk costs. Benefits were derived from sale of hydroelectric power, flood mitigation damage reduction, and from water supply. Because comparisons were to be made between benefits computed with and without the benefit of a forecast, operation and maintenance costs were not included. It was further assumed that flood benefits could be indexed to reservoir freeboard; all controlled releases and generated power were assumed to be purchased at constant prices. In this example penalty functions were not included.

While the resultant objective function was simplistic, it did provide an opportunity to explore the relative utility of a forecast as well as to determine the feasibility of developing operating rules when conditional flow distributions were explicitly included. An extremely important part of the entire analysis effort involved determining these conditional distributions. Monthly distributions of flow conditioned upon a forecasted cumulated flow volume for the next n months could be determined in a relatively straightforward fashion if the conditional distributions were unskewed. This, unfortunately, is rarely the case; forecasted flows were therefore disaggregated using the method of Chapter 3. The method is again summarized here for completeness so that it will be clear how forecasted flows are incorporated into the optimization model which is used to show some relative advantages of flow forecasts when developing reservoir release policies.

4.2 Probabilistic Flow Forecasts

Properties of conditionally distributed flows are illustrated using flow data for the Cedar River, Washington. (All flow data reported here are for the Cedar River.) The forecast period under study was the 6 months from March to August. All relevant model parameters were obtained from a 55-year (1914 - 1969) flow record. Normal or 3PLN distributions were force fitted to raw data of monthly and seasonal flow sequences via quantile-quantile plots (Wilk and Gnanadesikan, 1968) to remove the effects of a few large monthly flows on sample skews. The unconditional first three moments of mean, covariance, and skew were calculated for the smoothed data. Smoothed sample skews were modified using bias correction factors developed by Wallis et al. (1974), and Bobée and Robitaille (1975). Table 1 shows summary statistics of the first three moments for these smoothed historical data. Flow volumes in months April and May were approximately normally distributed. Table 2 shows the conditional first three moments of monthly flows resulting from a specified seasonal flow forecast. It is clear that flow forecasts provide a reduction in monthly variances. For example, considerable variance reduction (48.4%) is seen in June. Table 3 shows an unconditional correlation matrix between monthly flows, and monthly and seasonal flows. Inspection of this correlation matrix indicates that the correlations between monthly and total seasonal flows (shown in the second to last column of Table 3) are larger than the intermonth correlations because of positive correlations between monthly flows. If a forecast was to be made by a time series approach (for example, autoregressive methods) at the end of February, the forecasted distributions a few months ahead approach the unconditional distributions because the product of intermonth correlations rapidly tends to be zero. Disaggregation of forecasted volumes is suited to the situations where correlation structures between

Table 1. Unconditional First Three Moments of Flow, Cedar River, Washington

Month	Mean (AF)	St.De (AF)	Skew
March	50622	11940	0.697
April	55295	11537	0.000
May	58553	14933	0.000
June	45678	18850	0.814
July	24691	9840	1.311
August	16796	4931	0.023
Total	251636	52115	0.279

Table 2. Conditional First Three Moments of Monthly Flows (March-August), Cedar River, Washington, When the Total Seasonal Runoff is Specified

Month	Mean (AF)			St.De (AF)	Skew
	30th*	70th	90th		
March	47770	53474	57622	10615	0.954
April	51019	59571	65789	8119	-0.303
May	52154	64952	74256	8517	-0.864
June	37258	54099	66343	9733	4.637
July	20768	28614	34318	6345	4.423
August	15481	18111	20023	4238	-0.022
Total	224450*	278821	318351		

* Forecasted flow is equal to the 30th percentile of the distribution of total seasonal (March-August) flow

Table 3. Unconditional Flow Correlation Coefficient Matrix, Cedar River, Washington

	Mar.	Apr.	May	Jun.	Jul.	Aug.	Total
March	1.000	0.412	0.182	0.101	0.194	0.131	0.458
April		1.000	0.635	0.380	0.296	0.208	0.711
May			1.000	0.666	0.449	0.286	0.821
June				1.000	0.807	0.467	0.856
July					1.000	0.474	0.764
August						1.000	0.511
Total							1.000

	Mar.	Apr.	May	Jun.	Jul.	Aug.
March	1.000					
April	0.138	1.000				
May	-0.384	0.127	1.000			
June	-0.634	-0.630	-0.127	1.000		
July	-0.272	-0.544	-0.486	0.459	1.000	
August	-0.135	-0.256	-0.273	0.067	0.151	1.000

Table 4. Conditional Flow Correlation Coefficient Matrix, Cedar River, Washington

monthly and seasonal flows reduce conditional variances. Table 4 contains the conditional correlation matrix between monthly forecasted flow volumes. The magnitudes of these conditional correlations are relatively large. Failure to incorporate these conditional correlations into a procedure for estimating conditional flow distributions is tantamount to throwing out valuable information. The method used herein incorporates this information.

The procedures followed to obtain conditional flow distributions for months March to August are briefly described.

1. Force fit normal or 3PLN distributions to the total seasonal flow volume and flow volume in each month.
2. Compute conditional means, covariances, and skews in the untransformed domain and transform them to the Gaussian domain using the approach given by Matalas (1967). Data from Tables 2 and 4 are used here.
3. Correlated transformed (i.e., normally distributed) variables are decomposed into mutually independent normal variables via the method of principal components.
4. Specific percentile values of these independent normal variables are inverse transformed to yield conditional disaggregated flow distributions.

We have tested the above procedure, described in mathematical detail in Chapter 3 and have found it to be quite satisfactory in practice. There are, of course, many pitfalls awaiting the neophyte who ventures into transformed domains. FORTRAN code (implementable on a CDC 6400 with IMSL software support) for effecting conditional disaggregation is given in Appendix 1.

Two examples of prbability distributions with and without forecasting are given in Figures 1 and 2. The numbers in these figures indicate the percentiles of the March-August flow forecast. Flow volumes corresponding to each percentile are shown in Tables 1 and 2. It is readily seen in Figures 1 and 2

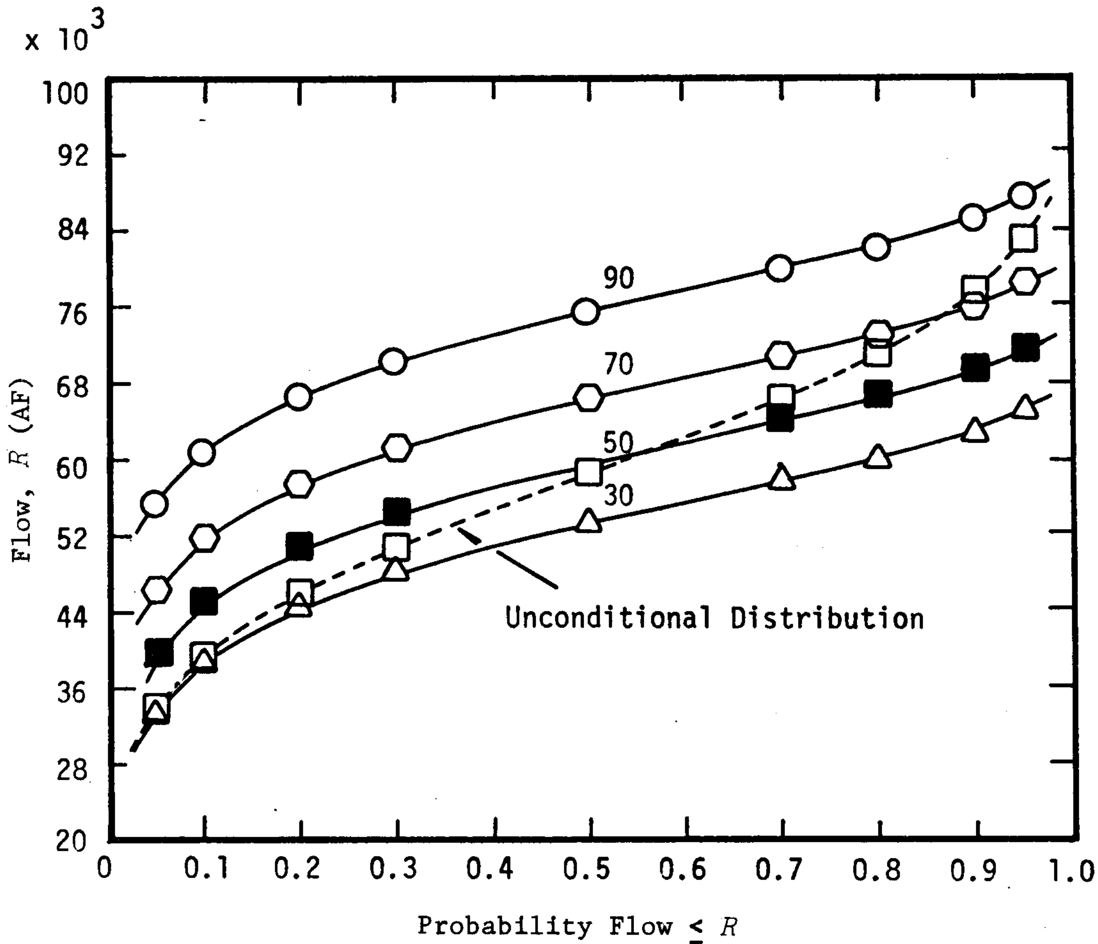

Figure 1. Cumulative Probability Distributions of Flow in May

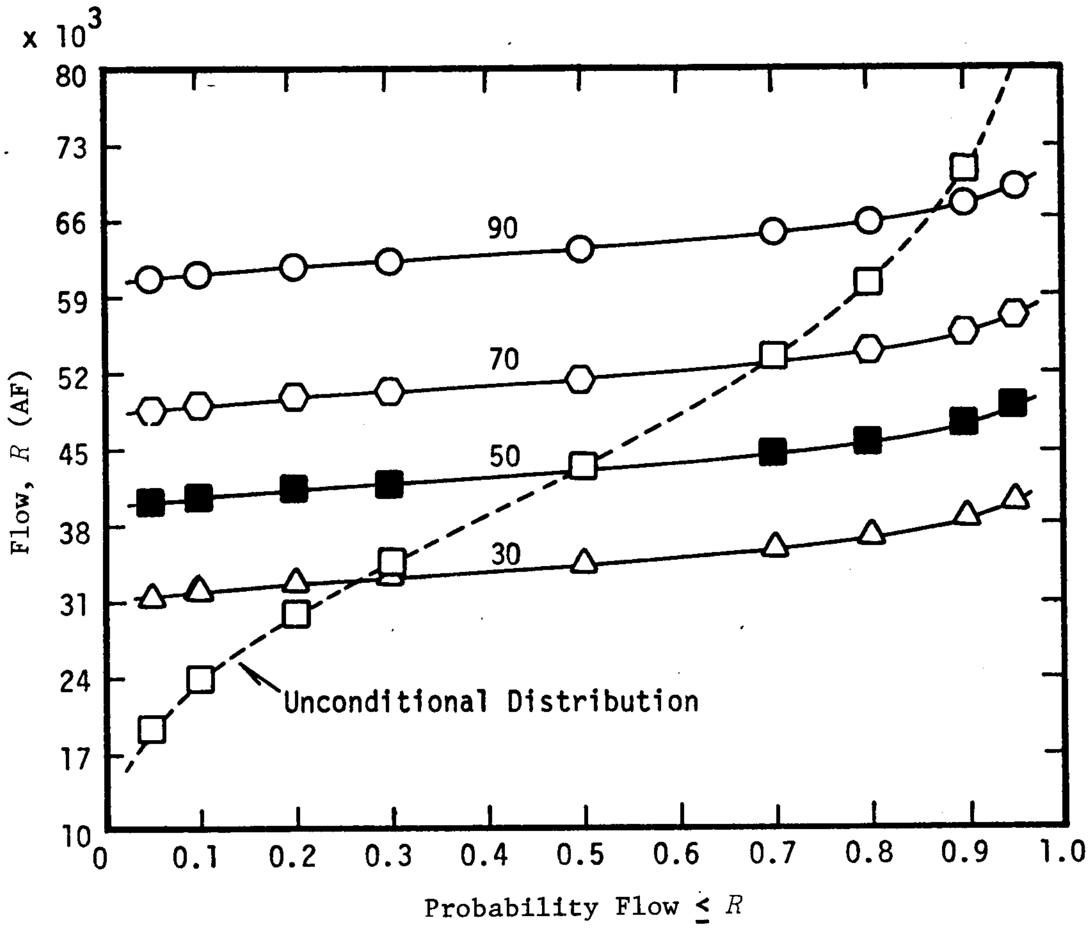

Figure 2. Cumulative Probability Distributions of Flow in June

that the range of forecasted inflows to a reservoir to be anticipated is remarkably less than that of the maximum ignorance (i.e., no forecast) case.

4.3 Worth of Flow Forecasts

Most water resource system operators can make use of information about future inflows to reservoirs. Total seasonal runoff volume forecasts made available sufficiently early in a year can be extremely useful for planning water allocation to potential users and uses. Forecasts have their greatest utility when the forecasted amount is less than what users have become accustomed to using. It is not clear how the true economic worth of a flow forecast can be evaluated. With sufficient notice, target releases from a reservoir can be modified, power contracts renegotiated, and the amount and types of crops to be irrigated can be appropriately planned. Some forms of insurance against supply shortfalls might be purchased by system users.

Estimation of the economic value of a forecast for a particular system would provide a very limited amount of information about the general utility of forecasting. Consequently, we elected to approach the issue by using a hypothetical single, multiple-purpose reservoir and examine theoretical operation policies for it. Operation policies were developed for both forecast and no forecast cases, all other parameters being held constant. Water supply, hydroelectric power generation, and flood mitigation benefits were considered. The difference in benefits computed for no forecast and forecast cases provides a measure of the worth of the forecast. The same benefit functions are used for both cases. Benefits from advance warnings of supply shortfall or surplus resulting from seasonal runoff forecasts were not calculated.

The purpose of this work is not to develop optimization models, but to assess the impact of improved forecast of streamflows on the operating policy of a single reservoir in the framework of economic benefits and losses.

The Linear Decision Rule (LDR) model proposed by Revelle et al. (1969), and extended by Houck (1975), was used to incorporate economic tradeoffs between flood control, water supply, and power production benefits. We do not advocate use of LDR's to operate a reservoir; they do, however, provide considerable information to assist with system operation. The advantages of the LDR approach, which led to adoption of the method for the comparisons made herein, are many. The LDR method is flexible and makes use of readily available linear programming (LP) software packages. The probabilistic nature of monthly streamflow is explicitly incorporated. (In the current work, Houck's method was extended to incorporate flow stochasticity.) The LDR is able to accommodate conflicting objectives, e.g., power generation, water supply, flood mitigation, etc. Finally, the LDR approach encodes risk preference explicitly and is within computational feasibility even for multiple-reservoir systems.

4.3.1 Chance Constrained LDR Formulation

The mathematical development given below basically follows Houck's (1975) approach. The equations are fully developed here for completeness. The fundamental premise in the LDR operating policy, originally developed by ReVelle et al. (1969), is that release X_t during period t is given by the difference between the initial storage S_{t-1} and the decision variable B_t, i.e.,

$$X_t = S_{t-1} - B_t \qquad \forall\, t \tag{4.1}$$

The continuity equation is

$$S_t = S_{t-1} + R_t - X_t \qquad \forall\, t \tag{4.2}$$

where R_t is the inflow for period t.

Substitution of (4.1) into (4.2) gives

$$S_t = R_t + B_t \qquad \forall\, t \tag{4.3}$$

By substituting the value of S_{t-1} from (4.3) into (4.1), the release, X_t, is

$$X_t = R_{t-1} + B_{t-1} - B_t \qquad \forall\, t \tag{4.4}$$

Equations (4.3) and (4.4) express storage at the end of any period and release during any period in terms of the decision variables, B_t, and the random variables of inflow, R_t, whose probability distributions are known. A set of requirements is imposed by probabilistic statements concerning the performance of reservoir functions.

Physical Constraints -- Two constraints limit the permissible range of reservoir storage volumes. The first is that the storage content will be no more than the reservoir capacity, C, with at least probability α_1, i.e.,

$$P[S_t \leq C] \geq \alpha_1 \qquad \forall\, t \tag{4.5}$$

Substituting (4.3) into (4.5) yields

$$C - B_t \geq F_t^{-1}(\alpha_1) \qquad \forall\, t \tag{4.5a}$$

where $F_t^{-1}(\alpha_1)$ is the inverse of the cumulative distribution function of inflow, R_t, during period t corresponding to the cumulative probability level α_1 (i.e., the $100*\alpha_1$ percentile).

The second constraint requires that storage will always exceed a minimum value, S_{min}, with a probability of at least α_2 in each period t

$$P[S_t \geq S_{min}] \geq \alpha_2 \qquad \forall\, t \tag{4.6}$$

The "deterministic" equivalent of (4.6) reduces to

$$S_{min} - B_t \leq F_t^{-1}(1-\alpha_2) \qquad \forall\, t \tag{4.6a}$$

Flood Control Benefit -- A valuable improvement in the LDR model proposed by Houck (1975) is the incorporation of economic benefits for flood control, water supply, and hydroelectric energy production.

A probabilistic formulation of the freeboard constraint is given by

$$P\left[C - S_t \geq FB_t^i\right] = \alpha_3^i \qquad \forall\ t \tag{4.7}$$

or

$$C - B_t - FB_t^i = F_t^{-1}\left(\alpha_3^i\right) \qquad \forall\ t \tag{4.7a}$$

where FB_t^i is a variable freeboard which is available $100*\alpha_3^i$ percent of the time during period t. Total freboard is broken into L increments. This is done to permit computation of expected flood mitigation benefit. The set of α_3^i (i = 1, 2,...., L) must satisfy the following requirements.

$$\alpha_3^{i-1} \leq \alpha_3^i \tag{4.8}$$

$$\sum_{i=1}^{L}\left(\alpha_3^i - \alpha_3^{i-1}\right) = 1 \tag{4.9}$$

$$\alpha_3^0 = 0, \qquad \alpha_3^L = 1 \tag{4.10}$$

The average freeboard available in the range between FB_t^{i-1} and FB_t^i can be approximated by $\left(FB_t^{i-1} + FB_t^i\right)/2$ with a probability of $\left(\alpha_3^i - \alpha_3^{i-1}\right)$, so that the expected benefit of flood control (BFC_t) in period t can be described by

$$BFC_t = \sum_{i=1}^{L}\left(\alpha_3^i - \alpha_3^{i-1}\right) * \eta * \left(FB_t^{i-1} + FB_t^i\right)/2 \qquad \forall\ t \tag{4.11}$$

where η is the benefit per unit volume of freeboard, and

$$FB_t^0 = C, \qquad FB_t^L = 0 \tag{4.12}$$

The benefit function is assumed to be a linear function of freeboard volume; nonlinearities can, however, be incorporated into the model by piecewise linearization. Equation (4.12) indicates that the freeboard available 0 percent of the time in period t equals the storage capacity; freeboard which is available 100 percent of the time is zero since the dam has a finite capacity and can spill water.

Water Supply Benefit -- The constraint on water supply can be expressed as

$$P\left[X_t \geq WS_t^i\right] = \alpha_4^i \qquad \forall\ t \tag{4.13}$$

Using (4.4), the "deterministic" equivalent of (4.13) reduces to

$$WS_t^i - B_{t-1} + B_t = F_{t-1}^{-1}\left(1-\alpha_4^i\right) \qquad \forall\ t \tag{4.13a}$$

where WS_t^i is a variable water supply which is available $100*\alpha_4^i$ percent of the time in period t. The probability levels of α_4^i $(i = 1, 2, \ldots, M)$, analogous to α_3^i for flood control, must satisfy the following requirements.

$$\alpha_4^{i-1} \leq \alpha_4^i \tag{4.14}$$

$$\sum_{i=1}^{M}\left(\alpha_4^i - \alpha_4^{i-1}\right) = 1 \tag{4.15}$$

$$\alpha_4^0 = 0, \qquad \alpha_4^M = 1 \tag{4.16}$$

The expected benefit of water supply (BWS_t) in period t can be approximated by

$$BWS_t = \sum_{i=1}^{M}\left(\alpha_4^i - \alpha_4^{i-1}\right) * \theta * \left(WS_t^{i-1} + WS_t^i\right)/2 \qquad \forall\ t \tag{4.17}$$

where θ is the net benefit per unit volume of water supply, and

$$WS_t^0 = F_{max}, \qquad WS_t^M = 0 \tag{4.18}$$

Equation (4.18) states that the water supply available 0 percent of the time in period t must equal the maximum allowable release (F_{max}) and water supply available 100 percent of the time is zero.

Power Production Benefit -- Hydropower production is a quadratic function (head multiplied by release through the turbines). This nonlinear relationship poses significant programming difficulties within the LDR formulation. To render the problem tractable in the LDR method, Houck (1975) introduced an approximate scheme for the expected value of power produced in period t

$$E(PR_t) = c * E(H_{t-1} * X_t) \qquad \forall t \tag{4.19}$$

where $E(\cdot)$ denotes the expectation operator, PR_t is electric energy produced in Kw-hr, c is the product of a conversion factor (CF) and the total efficiency of the power plant (e), H_{t-1} is head in feet, and X_t is release in acre-feet (AF). If head is assumed to be a linear function of storage volume (in many reservoirs this is a good approximation over usually experienced operating heads), the expected value of head and release is

$$E(H_{t-1} * X_t) = \sigma_{H_{t-1}} * \sigma_{X_t} + E(H_{t-1}) * E(X_t) \tag{4.20}$$

where

$$H_t = a * S_t + b \tag{4.21}$$

$$\sigma_{H_{t-1}} = a * \sigma_{R_{t-1}} \tag{4.22}$$

$$\sigma_{X_t} = \sigma_{R_{t-1}} \tag{4.23}$$

$$E(H_{t-1}) = a * E(R_{t-1}) + a * B_{t-1} + b \tag{4.24}$$

$$E(X_t) = E(R_{t-1}) + B_{t-1} - b_t \tag{4.25}$$

where σ_z is the standard deviation of variable z. When the linear relationship of (4.21) is assumed, the correlation coefficient between H_{t-1} and X_t becomes unity. Equation (4.20) is still a nonlinear function of $E(H_{t-1})$ and $E(X_t)$. An approximate solution of (4.19), without destroying the linearity of the constraint, is given by

$$E(PR_t) - \left[c * \overline{E(H_{t-1})}/2\right] * E(X_t) - \left[c * \overline{E(X_t)}/2\right] * E(H_{t-1})$$
$$= c * \sigma_{H_{t-1}} * \sigma_{X_t} \qquad \forall\ t \tag{4.19a}$$

The LDR formulation therefore requires iterative procedures; assumed initial values of $\overline{E(H_{t-1})}$ and $\overline{E(X_t)}$ whose expressions are similar to (4.24) and (4.25) are required. However, in several experiments conducted by the authors, the initial assumptions proved satisfactory, the LDR solution converged on the first iteration (i.e., the decision variables B_t derived from the first iteration were identical with those from the second iteration). With different relative benefits for the various reservoir uses, this fortunate approximation may not result--more than one iteration may be required. Hydroelectric power production (BPP_t) in period t is

$$BPP_t = \lambda * E(PR_t) \qquad \forall\ t \tag{4.26}$$

where λ is the benefit per unit of hydroelectric power produced. A more complicated power sales (benefit) structure was not needed in this work.

<u>Objective Function</u> -- The objective function is to maximize the economic benefits resulting from flood control, water supply, and power production, i.e.,

$$\text{Maximize } Z = \sum_{t=1}^{n} BFC_t + \sum_{t=1}^{n} BWS_t + \sum_{t=1}^{n} BPP_t \tag{4.27}$$

where n is the number of months under consideration. Benefits for the months of the year where flow forecasts were not made were not included in (4.27).

4.3.2 Physical Parameters Used in the LDR Formulation

Potential improvements in economic benefit from operation of a given reservoir for a given level of improvement in inflow forecasts clearly depend in the reservoir size and its uses. Most reservoirs in use in the United

States, particularly those supplied by snowmelt runoff, are designed to smooth out seasonal flow fluctuations and can be categorized as within-year flow buffering facilities. Reservoirs whose capacity exceeds the mean annual flow (e.g., the major reservoirs on the Lower Colorado River) are able to provide flow buffering over several years. Improved forecast accuracy is usually of greater significance in less resilient systems, i.e., in systems having small capacity relative to the mean annual flow.

Based upon flow data from the Cedar River, Washington, two hypothetical reservoirs are examined, one having capacity approximately equal to 20% of the mean annual flow volume, and the other having a capacity approximately equal to the mean annual flow volume. Numerous cases were tested to determine the importance of facility capacity, initial storage, and the forecasted flow magnitude. Physical parameters used in the LDR solution are summarized below.

Reservoir Capacity -- C = 100,000 AF (20% of mean annual flow volume), required minimum storage: S_{min} = 20,000 AF, initial storage: (i) S_0 = 30,000 AF; (ii) S_0 = 80,000 AF, linear relationship between head (H_t; feet) and storage (S_t; AF): $a = 10^{-3}$ and $b = 50$.

Reservoir Capacity -- C = 500,000 AF (mean annual flow volume), required minimum storage: S_{min} = 200,000 AF, initial storage: S_0 = 450,000 AF, linear relationship between head (H_t; feet) and storage (S_t; AF): $a = 5 * 10^{-3}$ and $b = 50$.

Other Parameters -- Maximum allowable release: F_{max} = 90,000 AF, coefficient of power production function: $c = 0.72$.

Probability Levels -- The probability levels used in the LDR formulation are arbitrarily chosen to reflect the system owner's risk preference. Operating results are influenced by the choice of the several α's. It has been pointed out by Loucks and Dorfman (1975) that the setting of fixed chance constraint levels is conservative. Hence, if say a 95% level was set in the model, a 99% level might be realized when the system is operated under the derived LDR's for simulated inflow scenarios. This results from modeling probabilistic and not stochastic inflow. Despite these criticisms the following probability levels were used; reservoir capacity constraint: $\alpha_1 = 0.90$, minimum storage constraint: $\alpha_2 = 0.90$, variable freeboard and water supply constraints ($L = M = 6$):

i	0	1	2	3	4	5	6
α_3^i , α_4^i	0.0	0.1	0.3	0.5	0.7	0.9	1.0

Benefit Functions -- Benefit for freeboard: η = \$2/AF, benefit for water supply: θ = \$10/AF, benefit for hydroelectirc power production: λ = \$0.02/Kw-hr.

Parameters, probability levels, and benefit functions described above are identical in the LDR formulation for both reservoir capacities (0.2 μ and 1.0 μ) examined.

4.4 Numerical Results from Chance Constrained Reservoir Operation Formulation

In all cases the chance constrained LP's were solved using MPOS (Multiple Purpose Optimization System) software packages implemented on a CDC 6400 digital computer at the University of Washington, Academic Computer Center. The formulations for the six-month period of interest (March to August) required 104 variables and 106 constraints. Benefits were computed in three different ways; for convenience they are referred to as Types A, B, and C defined below.

Type A: LDR algorithm was solved using unconditional marginal distributions of monthly inflow volumes and the benefits associated with flood control, water supply, and power production were computed. [This is the approach taken by Houck (1975).]

Type B: Economic returns resulting from flood control, water supply, and power production were computed using decision variables based on unconditional flow distributions, i.e., LDR's were determined as for Type A benefits; actual benefits were, however, computed using conditional flows while the reservoir was operated with these LDR's.

Type C: LDR's were developed by solving the formulation for conditional (i.e., forecasted) inflows and the resulting benefits determined.

Forcast volumes ranging between the 30th and 90th percentiles of March-August total runoff volume were used to compute Type C benefits. (Tables 1 and 2 show the magnitudes of these flow volumes.)

4.4.1 Reservoir Capacity 20% of Mean Annual Flow Volume

Table 5 shows optimal values of the decision variables B_t depending on knowledge of inflow volumes for a reservoir capacity of 100,000 AF; different initial storage conditions of 30,000 AF and 80,000 AF were used. In Table 5a, for instance, LDR's for unconditional flows are given in the third row, indicated as scheme A. The LDR's given in the fourth row, indicated as scheme C(30), are those rules determined by solving the problem with flows conditioned by a forecast flow at the 30th percentile of total season runoff. Notice that for June the value of B_t changes from -3806 AF for the normally formulated LDR problem to -11949 AF when the forecast is at the 30th percentile level, to -41033 AF when the 90th percentile of total runoff

Table 5. Decision Variables, B_t (acre feet) Computed From Chance Constrained Formulation of Reservoir Operation Using Conditional and Unconditional Monthly Flows; Reservoir Capacity: C = 100,000 acre feet

(a) S_o = 30,000 acre feet (AF)

Scheme	Decision Variable (AF)					
	March	April	May	June	July	August
A #	-16530	-20526	-19437	-3806	5882	9504
C(30)*	-17491	-10526	-18634	-11949	4267	4775
C(50)	-20343	-14803	-25033	-20369	344	3460
C(70)	-23195	-19079	-31431	-28789	-3579	2145
C(90)	-27342	-25296	-40736	-41033	-9283	233

(b) S_o = 80,000 acre feet (AF)

Scheme	Decision Variable (AF)					
	March	April	May	June	July	August
A #	20000	-20526	-19437	-3806	5882	9504
C(30)*	20000	-10526	-18634	-11949	4267	4775
C(50)	20000	-14803	-25033	-20369	344	3640
C(70)	20000	-19079	-31431	-28789	-3579	2145
C(90)	20000	-25296	-40736	-41033	-9283	233

\# Unconditional distribution

* Conditional distribution when the 30th percentile of the distribution of total seasonal flow is specified

is forecasted. Note that for the same inflow information, different initial storages yield identical release decision variables except in March. (When the LDR follows a regular cyclic pattern in a year, 12 monthly decision variables are the same from year to year and therefore are independent of the initial storage employed. The initial storage was specified in the LDR formulation here because the forecast time period was for 6 months of the year.) While the release rules are virtually independent of the initial storage, the benefits (Table 6) are influenced by the initial state.

Figures 3 and 4 show the computed freeboards $\left(FB_t^i\right)$ corresponding to each reliability level, α_3^i , in May and June, respectively, for a reservoir capacity of 100,000 AF and initial storages of 30,000 AF and 80,000 AF. It is clear from these figures that Type C (forecast distribution) operation gives rise to larger freeboards at every probability level than does Type A (unconditional distribution). From a practical viewpoint, it is important to know the freeboard available at the higher reliability level. For example, the difference of freeboards between Type C and Type A operation at the 90% reliability level is 13,800 AF in May. This increased amount yielded from Type C operation indicates that 30% more storage is used for flood mitigation than is used under Type A operation. For the month of June, the difference between the two types of operation yields 40,200 AF. It is important to state here that the freeboard available at each probability level in Type C is the same even when changing the percentile levels of the March-August flow forecast. This characteristic is explained by the fact that the difference between two decision variables for different forecast levels of seasonal runoff is identical with that between the corresponding conditional means in each month. For example, in May the difference in decision variables between C(50) and C(30) [or C(70) and C(50)] is 6,400 AF, which is identical to the differences

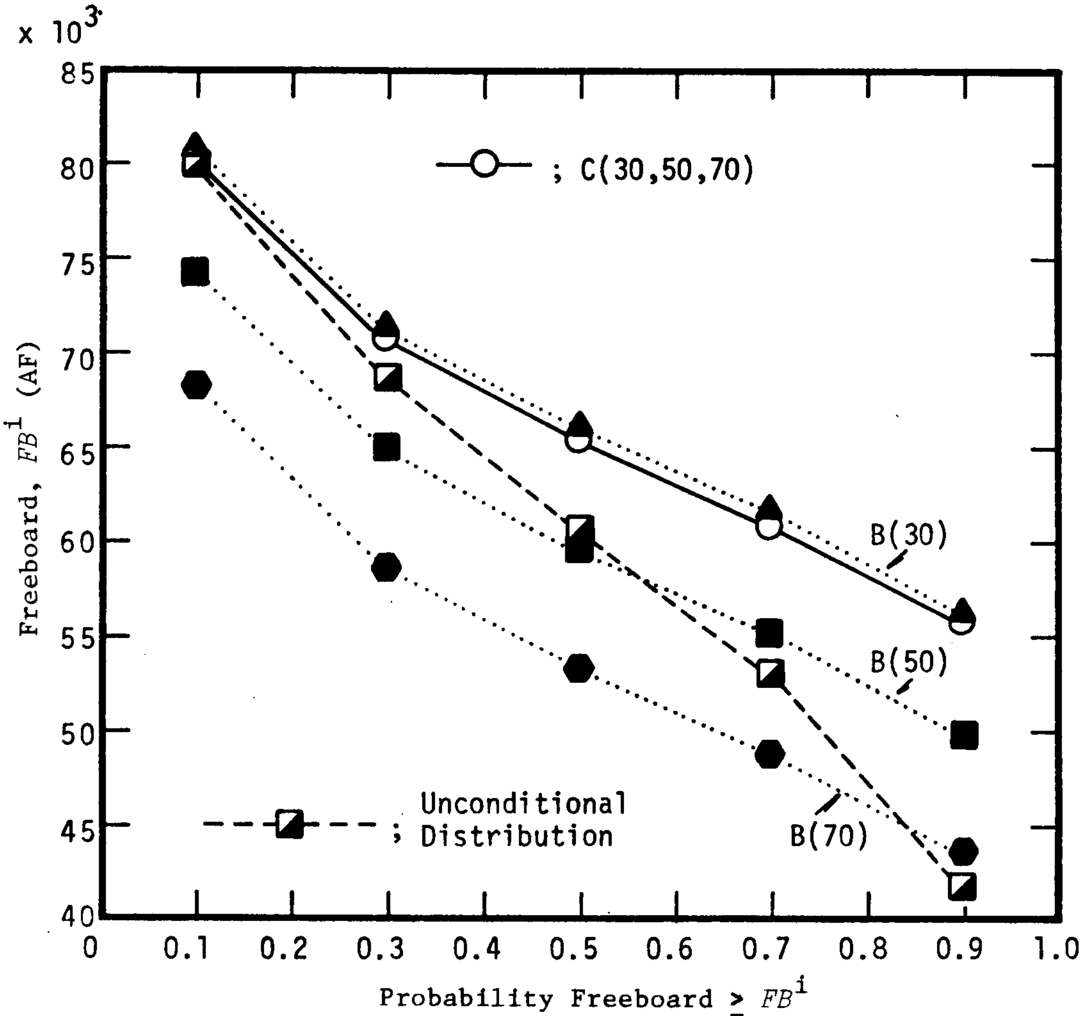

Figure 3. Cumulative Probability Distribution of Freeboard in May (Capacity = 100,000 AF, S_o = 30,000 and 80,000 AF)

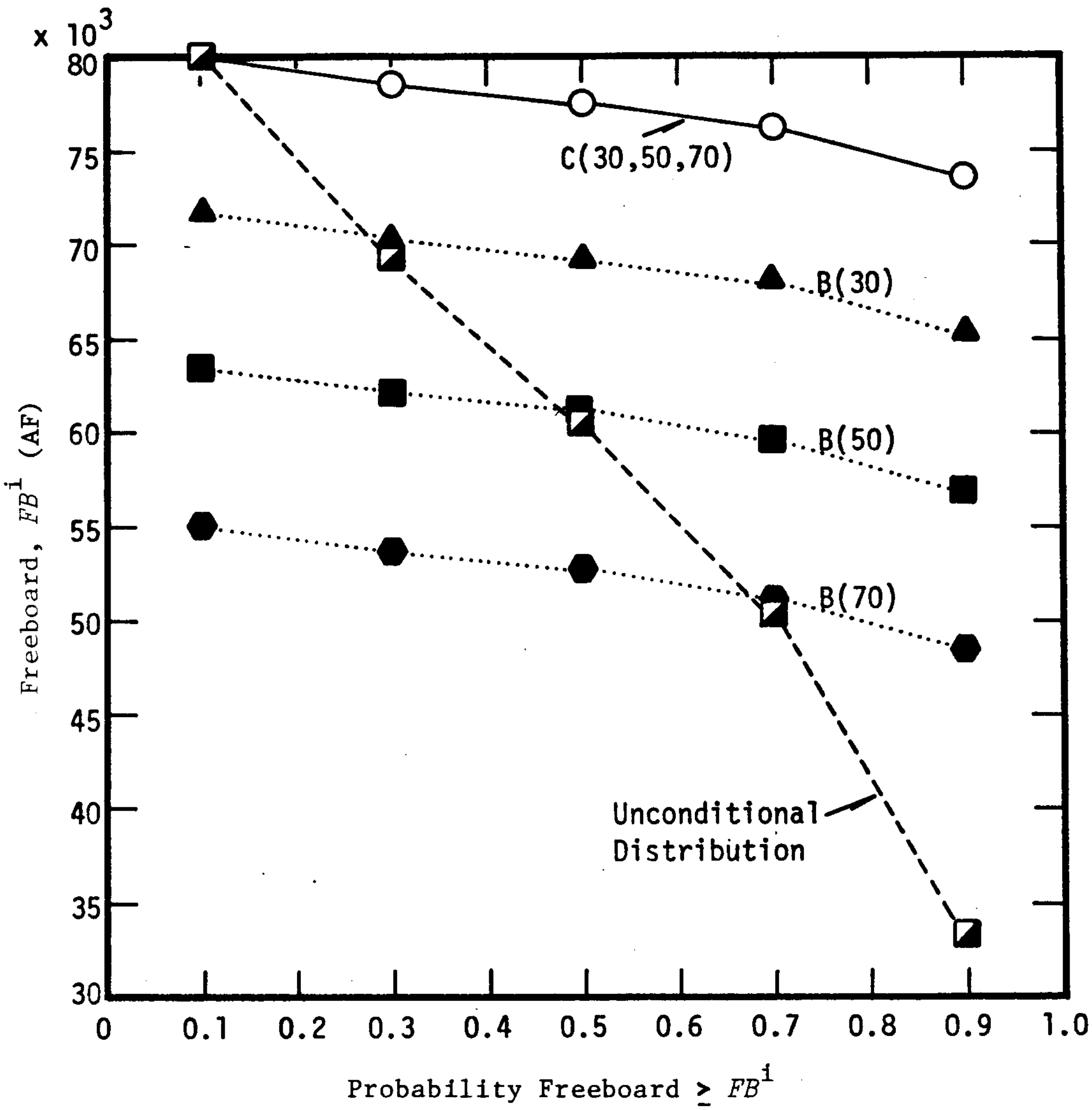

Figure 4. Cumulative Probability Distribution of Freeboard in June (Capacity = 100,000 AF, S_o = 30,000 and 80,000 AF)

between the conditional means in May (Tables 1 and 2). As shown in Figures 1 and 2, any desired conditional distribution in a given month is derived by displacing any of the conditional distribution curves downward or upward by the difference between two conditional means. Hence, the freeboard $\left(FB_t^i\right)$ expressed in (4.7a) is constant with respect to $C - [B_t + F_t^{-1}(\alpha_3^i)]$ for different forecasted levels of March-August flow. A similar observation can be made in the freeboard curves resulting from Type B operation in Figures 3 and 4. A particular freeboard curve is translated downward or upward by the difference between the conditional means. Flow/operation condition B(30) corresponds with the time of low flow volumes to be anticipated in which the LDR solutions were based upon unconditional flows, but economic returns were evaluated for conditional inflows using the 30th percentile forecast of the total seasonal runoff. Type B tends to yield larger freeboards at relatively high reliabilities than Type A even when high inflow volumes [B(70)] are to be expected. It is of interest to compare Figures 2 and 4. The smaller range of forecast probability distribution than that of the unconditional marginal flow distribution in Figure 2 gives rise to a smaller range of freeboard curves for Type B and Type C operation than for Type A operation. Figures 1 and 3 yield similar observations.

Figure 5 shows cumulative distributions of water supply for a reservoir having capacity of 100,000 AF, S_0 = 30,000 and 80,000 AF. Larger water supplies can be satisfied under Type C operation than for Type A operation, reflecting lesser uncertainty in the range of flow in each month when forecasts are made. A 34% larger supply can be made in June at the 90% reliability level when Type C [C(30)] instead of Type A operation is used. Similar observations were made in other months in the March-August period. Note that C(30) reflects a lower-flow state than average runoff volume, yet increased supply

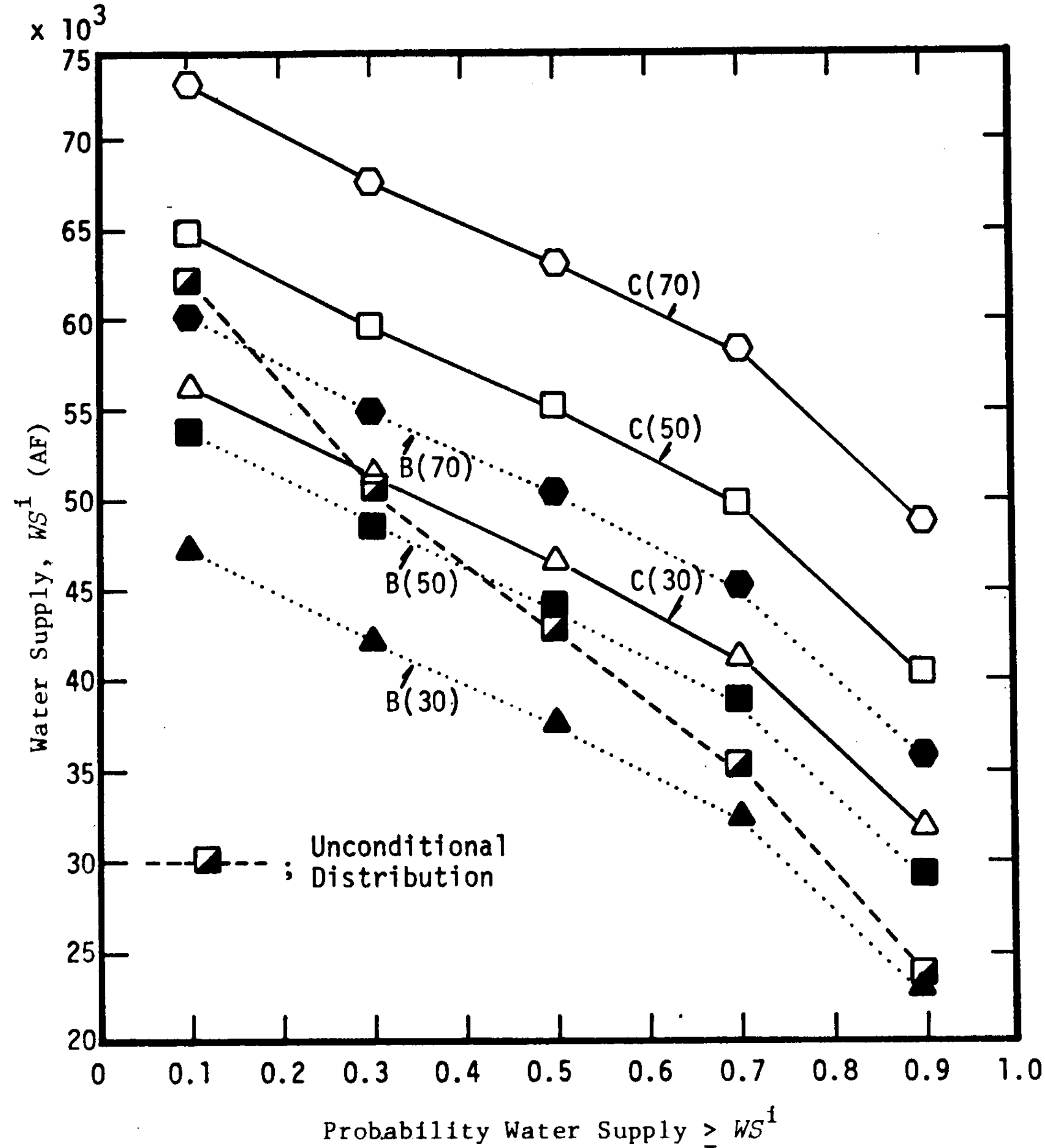

Figure 5. Cumulative Probability Distribution of Water Supply in June (Capacity = 100,000 AF, S_o = 30,000 and 80,000 AF)

above the no forecast case can be made. Water supply curves of Type B and Type C in Figure 5 are parallel with each other as for freeboard curves in Figures 3 and 4; the difference between two Type C water supply curves is identical with that between two conditional means in June (Tables 1 and 2). Similarly, the difference between two Type B water supply curves in Figure 5 is identical with that between two conditional means in May, by (4.13a). As shown in Table 5, the decision variables in May and June, respectively, are the same for identical probability distributions of flow regardless of the initial storage levels; the freeboard and water supply curves in these two months are independent of the initial storages assumed. While not shown, freeboard curves in March, and water supply curves in March and April, are influenced by the initial storage state.

Table 6 shows quantitatively the conflicting nature of the demands placed upon a multiple-purpose reservoir; the economic significance of the total seasonal forecast is clear. The following comparisons serve to emphasize these conflicts. B(30) and C(30) represent inflow volumes below normal (unconditional means). Hence, it is expected that benefits from flood control for Type B and Type C operation are larger than those for Type A operation, while benefits from water supply and power production are smaller. The total benefit for 6 months becomes larger as high inflow volumes are to be anticipated. For example, if C(70) and an initial storage of 30,000 AF occurred, the conditional operation rule yields an 11% increase in total benefit compared with the average chance constrained Type A operation. A 23% increase occurs for C(90). Types B and C operations use identical conditionally distributed flows; Type B operation uses Type A derived LDR's, however. It is clear from Table 6 that Type C economic benefits yield larger total benefits than economic returns of Type B. An important observation in Table 6 is that for a

Table 6. Benefits From Reservoir Operation Using Conditional and Unconditional Flows and Operating Rules; Reservoir Capacity: C = 100,000 acre feet

(a) S_o = 30,000 acre feet (AF)

Scheme	Flood Control ($)	Water Supply ($)	Power Production ($)	Total ($)
A	771282	2302692	326739	3400714
B(30)	823075	2055366	273155	3151596
B(50)	774141	2288201	318608	3380950
B(70)	725208	2521033	368488	3614729
B(90)	654054	2859592	448913	3962559
C(30)	831436	2098888	283885	3214208
C(50)	831437	2346410	316252	3494099
C(70)	831435	2593929	348624	3773987
C(90)	831433	2953843	395693	4180969

(b) S_o = 80,000 acre feet (AF)

Scheme	Flood Control ($)	Water Supply ($)	Power Production ($)	Total ($)
A	705528	2766162	477639	3949329
B(30)	757321	2518836	421054	3697211
B(50)	708387	2751671	469507	3929565
B(70)	659454	2984503	522388	4166345
B(90)	588300	3323062	607177	4518539
C(30)	763952	2561397	427105	3752454
C(50)	758819	2806067	466841	4031728
C(70)	753684	3050734	507166	4311583
C(90)	746218	3406501	566845	4719564

small initial storage of 30,000 AF, Type C benefits from flood control are nearly constant, thus affording inflows to be stored. On the other hand, for a larger initial storage of 80,000 AF, benefits from flood control, as expected, decrease as inflow volumes are larger and the dam spills water.

4.4.2 Reservoir Capacity Equal to Mean Annual Flow Volume

Another experiment was made to examine the importance of reservoir capacity. The results for a reservoir having capacity of 500,000 AF (approximately the mean annual flow) and an initial storage of 450,000 AF are shown below. Table 7 gives the decision variables for both unconditional marginal and conditional probability distributions of inflow volumes. The decision rules in Table 5 (Capacity = 100,000 AF) are mainly negative. The decision rules in Table 7 are all positive (Capacity = 500,000 AF), reflecting vastly different operating strategies for large and small reservoirs subject to the same patterns of inflow; equation (4.3) emphasizes the role of reservoir storage.

Figure 6 shows freeboard-reliability curves in May, and Figure 7 shows water supply-reliability curves in June. The behavior of these probability distributions for a capacity of 500,000 AF bears out similar conclusions for a capacity of 500,000 AF; the conditional LDR based on improved forecasts of inflow volumes (Type C) yields a much larger freeboard and water supply volume, particularly at higher reliability levels than the unconditional LDR's (Type A and Type B) do. For example, in Figure 6 the freeboard available 90% of the time can be increased by 5.3% in Type C, relative to Type A operation. Even though C(50) has the same mean flow as the unconditional mean, it is shown in Figure 7 that changing from Type A to Type C operation increases the water supply volume by 69.6% at the 90% probability level in June. Figure 7 shows a range of water supply for C(50) between about 42,000 and 65,000 AF.

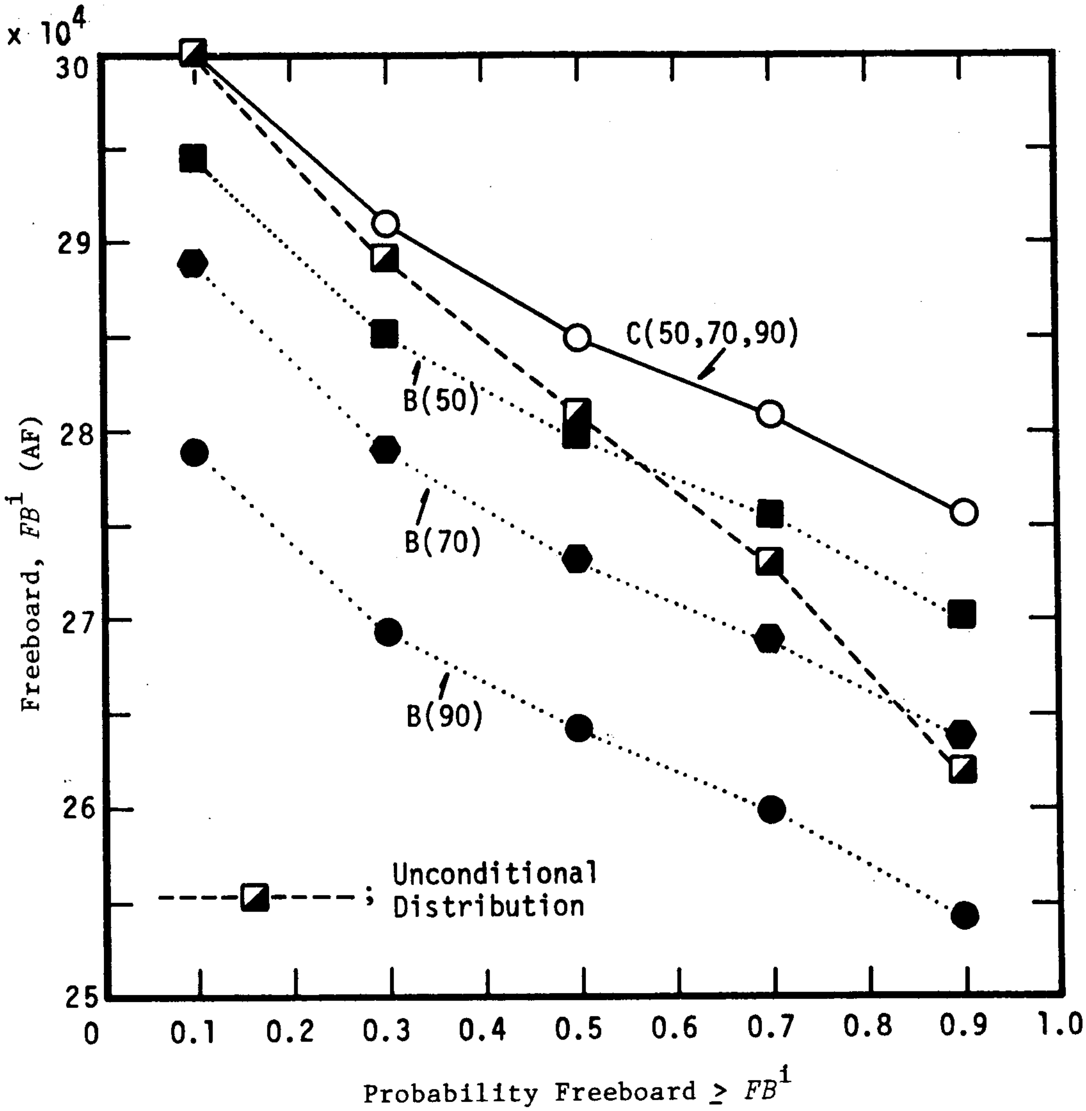

Figure 6. Cumulative Probability Distribution of Freeboard in May (Capacity = 500,000 AF, S_o = 450,000 AF)

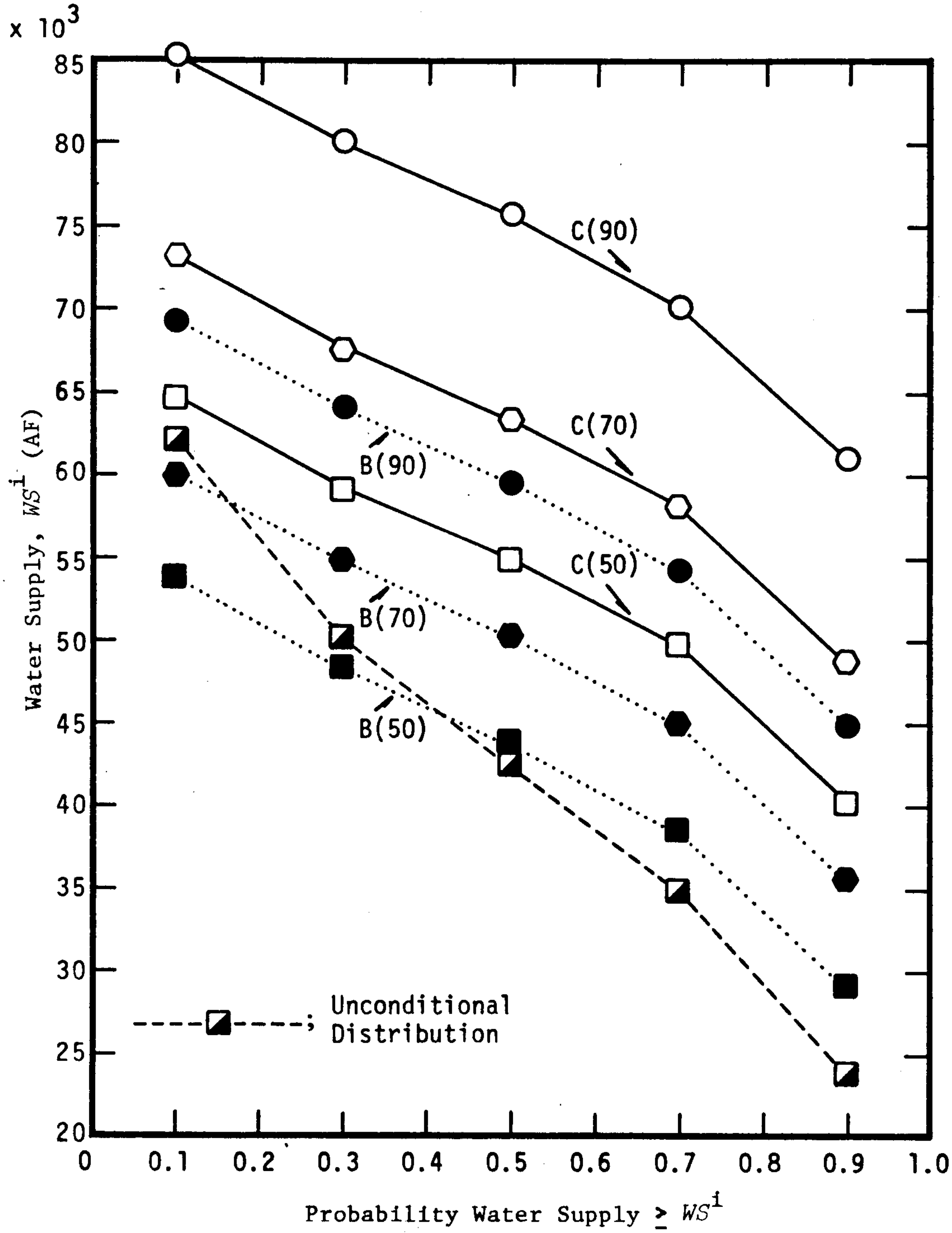

Figure 7. Cumulative Probability Distribution of Water Supply in June (Capacity = 500,000 AF, S_0 = 450,000 AF)

The same range for Type A operation is between about 23,000 and 63,000 AF. The latter situation necessitates less efficient allocation of resources. The flatter the curves in Figures 6 and 7 in each month, the easier it is to operate the system efficiently. Freeboard and water supply probability distributions determined in the above manner allow system operators to examine conflicting system performances explicitly, while attempting to meet a given operating schedule at a prescribed reliability level.

Table 8 shows benefits computed for a capacity of 500,000 AF and an initial storage of 450,000 AF. It is of interest to compare the results shown in Table 8 with those in Table 6b. Both cases have large initial storage relative to the reservoir capacity. Benefits accruing from flood damage mitigation under Type C operation for the larger capacity reservoir are almost identical for different forecast levels. In Table 6b, Type B economic returns are larger for power production than Type C benefits for the same flow conditions. In contrast, in Table 8, Type C operation yields larger benefits for power production as well as for flood damage mitigation and water supply than does Type B operation. Compared with Type A operation, Type C(70) operation yields a 4.5% increase in benefits in terms of the total benefit (9.2% in Table 6b), while C(90) achieves an increase of 9.5% (19.5% in Table 6b). Reservoir size, relative to the flow of a river, is important when determining the relative improvement in operation resulting from incorporation of a forecast of total seasonal runoff volume.

4.5 Summary and Conclusions

There are presently three basic ways to operate a multiple-purpose reservoir. The first utilizes fully the operator's experience and judgment. The second makes use of optimization methods and assumed streamflow and demand methods. The most useful approach, despite some criticisms that can be made,

Table 7. Decision Variables, B_t (acre feet), Computed from Chance Constrained Formulation of Reservoir Operation Using Conditional and Unconditional Monthly Flows; Reservoir Capacity: C = 500,000 acre feet, S_o = 450,000 acre feet (AF)

Scheme	Decision Variable (AF)					
	March	April	May	June	July	August
A #	390000	159395	160436	176194	185882	189504
C(50) *	390000	165197	154967	159631	180344	183460
C(70)	390000	160921	148569	151211	176421	182145
C(90)	390000	154704	139264	138967	170717	180233

\# Unconditional distribution

* Conditional distribution when the 50th percentile of the distribution of total seasonal flow is specified

Table 8. Benefits from Reservoir Operation Using Conditional and Unconditional Flows and Operating Rules; Reservoir Capacity: C = 500,000 acre feet, S_o = 450,000 acre feet (AF)

Scheme	Flood Control ($)	Water Supply ($)	Power Production ($)	Total ($)
A	2980085	4475231	1690850	9146166
B(50)	2982758	4461671	1686784	9131213
B(70)	2933824	4694503	1763859	9392187
B(90)	2862671	5033062	1879882	9775615
C(50)	3032819	4516067	1687577	9236463
C(70)	3027684	4760734	1766411	9554829
C(90)	3020218	5116501	1881567	10018286

is a chance constrained linear programming (CCLP) formulation of system operation. Here the experience and judgment of the operator can be incorporated explicitly via the chance constraints that are chosen. The CCLP is particularly suited for smaller reservoirs that are used for within-year flow buffering; it was the obvious choice to combine with seasonal runoff volume forecasts for snowmelt-fed rivers. While CCLP models have been used to develop linear decision rules (LDR's) for reservoir release, they have not been used previously with other than unconditional probability distributions of flow. It is possible, as shown here, to formulate the problem using flow distributions conditioned on the total forecasted amounts.

Using the CCLP formulation and conditional flow distributions, it was possible to show the general utility of incorporating the forecast explicitly into the operating procedure. While a limited situation was examined, and only one level of chance constraints was used, it appears that this type of forecast is extremely useful. There is, of course, great difficulty in indexing flood mitigation damage benefits to reservoir freeboard; however, the relative worth of operating with and without forecasts of future inflow is clear. A further advantage of this approach is that for a given reservoir capacity, chance constraints, and benefit functions, the problem need only be solved for one forecast condition. The cumulative distributions of freeboard and water supply can be readily determined for any other forecast by simply displacing the curves in each time period by an amount equal to the change in the conditional mean given the new forecast.

The method can be extended to multiple sites with the number of variables and constraints increasing approximately linearly with the number of sites. Obviously, considerable effort must be expended at a given location to determine the benefit function for each use of the system. In special situations, e.g., drought, more emphasis might be placed on one particular use. Here it would be possible to obtain very useful operating rules; the benefits would be crisply defined.

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APPENDIX 1: LISTING OF FORTRAN IV PROGRAM USED TO COMPUTE CONDITIONAL FLOW MARGINAL PROBABILITY DISTRIBUTIONS

```
      PROGRAM SUMSEA(INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7=P
     1UNCH)
C**   FORECAST OF MONTHLY FLOW AT LANDSGURG IN THE CEDAR RIVER
C**   FORECAST BY DISAGGREGATION MODEL FROM MARCH TO AUGUST
C**   INPUT DATA = CUBIC FEET PER SEC. DAYS
C**   HISTORICAL DATA AVAILABLE FROM OCT 1914 TO SEP 1969 (55 YRS)
      DIMENSION NFLW(13,65),AVER(13),STAN(13),SKEW(13),CORRE(12,12),
     1PCOE(12,12),BINV(12,12),A3(12,12),FLOW(13,65),XNAME(13,3),
     2SEASON(30),AVLOG(13),STLOG(13),AALOG(13)
      DIMENSION A4(20,20),A5(20,20),GAVE(13),GSTN(13),GSKW(13),
     1COVAR(12,12),AUTO(65),DIFF(12,12),RHS(65)
      DIMENSION CUMPRO(15),YINI(10),CDFX(15),PFLOW(13,15),COML(13),
     1PDIS(13,15),PERCET(15),ORAV(13),ORST(13)
      DIMENSION PBT(12,12),ACOE(12),TITLE(20),LU(13),LUD(13)
      DIMENSION AAB(78),Z(12,12),WK(90),ROOT(12),EIGEN(12)
      DIMENSION IODER(65)
C**     DIMENSION Y(2000),SEFLOW(6,2000)
C**    IF(NSIM = 1) Y(2000),SEFLOW(6,2000) ARE REQUIRED
      DIMENSION Y(100),SEFLOW(6,100)
      REAL NFLW
      EXTERNAL VIPDA
      READ(5,7700) NLAST,NDATA,NFLAG,NPRO,NLOG,NCARD,NSIM
      READ(5,7700)  NNIN
      READ(5,101) (CUMPRO(I),I=1,NPRO)
      READ(5,101) (YINI(I),I=1,NNIN)
      READ(5,7705) (TITLE(I),I=1,20)
      NMON=NLAST+1
      NSON=NLAST-1
      NHAF=NPRO/2
      NHAF1=NHAF+1
      DO 7701 I=1,NLAST
 7701 READ(5,7702) (NFLW(I,J),J=1,NDATA)
      DO 7703 I=1,NMON
 7703 READ(5,7704) (XNAME(I,J),J=1,3)
      IF(NSIM.EQ.0) GO TO 10001
      READ(5,7700) NTEST,NDATB
10001     CONTINUE
      IF(NFLAG.NE.1) GO TO 7842
      WRITE(6,6250) (TITLE(I),I=1,20)
      WRITE(6,6193) NLOG
 7842 CONTINUE
C***************
C**
C**    NLAST = NO. OF SEASONS
C**    NDATA = NO. OF OBSERVATIONS
C**    IF(NFLAG = 1) PRINT THE RESULTS
C**    NPRO = NO. OF PROBABILITY LEVELS IN THE DISTRIBUTION
C**   IF(NLOG = 0) NORMAL DISTRIBUTION
C**   IF(NLOG = 1) 3PLN DISTRIBUTION
C**    IF(NCARD = 1) CARD PRINT
C**    IF(NSIM = 1) GENERATE SEASONAL FLOWS WHEN CONDITIONAL MEAN
C**                 AND CONDITIONAL VARIANCE-COVARIANCE ARE GIVEN
C**    NNIN = NO. OF INITIAL VALUES IN AGGREGATE FLOW
C**    CUMPRO(I) = PROBABILITY LEVELS (0.05, 0.10, 0.20,-----,0.90,0.95)
C**    YINI(I) = PROBABILITY LEVEL OF INITIAL VALUE IN AGGREGATE FLOW
C**             (0.20, ----,0.80)
C**    NFLOW(I,J) = OBSERVED SEASONAL FLOWS
```

```
C**      XNAME(I,J) = SEASON NAME
C**      NTEST = NLAST
C**      NDATB = DESIRED NUMBER OF GENERATED SEASONAL FLOWS
C**             NDATB = 2000
C***************
C**
C**     COMPUTE THE FIRST THREE MOMENTS OF OBSERVED MONTHLY FLOWS
        KCH=0
        CALL MOMENT(NLAST,NDATA,NFLW,AVER,STAN,SKEW,KCH)
C**       CORRECT THE OUTLIERS IN BOTH TAILS BY SETTING THEM ECUAL TO THE
C**     THRESHHOLD VALUE (=MEAN PLUS AND MINUS 2.6*STINDARD DEVIATION)
        DO 7850 I=1,NLAST
        B=AVER(I)+2.6*STAN(I)
        DO 7851 J=1,NDATA
        A=NFLW(I,J)
        IF(A.LT.B) GO TO 7844
        NFLW(I,J)=B
        GO TO 7851
 7844   CONTINUE
 7851   CONTINUE
 7850   CONTINUE
        NFLW(1,2)=NFLW(1,18)=43121
C**      CONVERT FLOWS TO ACRE - FT (1.98)
        DO 7852 I=1,NLAST
        DO 7853 J=1,NDATA
        NFLW(I,J)=1.98*NFLW(I,J)
 7853   CONTINUE
 7852   CONTINUE
C**     COMPUTE THE AGGREGATE FLOW FROM CORRECTED MONTHLY FLOWS
        DO 7800 J=1,NDATA
        S=0.
        DO 7801 I=1,NLAST
 7801   S=S+NFLW(I,J)
        NFLW(NMON,J)=S
 7800   CONTINUE
        DO 7720 I=1,NMON
        DO 7720 J=1,NDATA
        FLOW(I,J)=NFLW(I,J)
 7720   CONTINUE
C**     COMPUTE THE FIRST THREE MOMENTS OF CORRECTED AND
C**     AGGREGATE FLOWS
        CALL MOMENT(NMON,NDATA,FLOW,AVER,STAN,SKEW,NFLAG)
        ANNAV=AVER(NMON)
        DO 7322 I=1,NMON
        IF(SKEW(I).LE.0.0) GO TO 7321
        LU(I)=1
        GO TO 7322
 7321   LU(I)=0
 7322   CONTINUE
C**     COMPUTE A CORRELATION MATRIX OF MONTHLY AND AGGRAGATE FLOWS
C**     IN THE REAL DOMAIN
        CALL SECOR(NMON,NDATA,NFLW,COVAR)
        IF(NFLAG.NE.1) GO TO 7803
        WRITE(6,6195)
        DO 7805 I=1,NMON
 7805   WRITE(6,205) (COVAR(I,J),J=1,I)
 7803   CONTINUE
```

```
C**    SKEWED FLOWS ARE NORMALIZED BY THE 3PLN DISTRIBUTION
       CALL TRANS(NMON ,NDATA,AVER,STAN,SKEW,FLOW,AVLOG,STLOG,AALOG,NFLAG
      1)
C**
C**      SKEWED FLOWS ARE FORCED TO BE DISTRIBUTED AS THE 3PLN DISTRI.
C**      BY USING THEORETICAL FIRST TWO MOMENTS IN THE NORMAL DOMAIN
C**
       XNN=NDATA+1
       DO 7308 LL=1,NDATA
       XIN=LL
       D1=1.-XIN/XNN
       CALL PROLEL(D1,STNRM)
       RHS(LL)=STNRM
 7308  CONTINUE
       DO 7310 I=1,NLAST
       JP=LU(I)
       DO 7312 J=1,NDATA
 7312  AUTO(J)=NFLW(I,J)
       CALL ORDER(NDATA,AUTO,IODER)
       DO 7314 K=1,NDATA
       JO=IODER(K)
       D1=RHS(K)
       IF(JP.EQ.0) GO TO 7311
       S=STLOG(I)*D1+AVLOG(I)
       S=EXP(S)+AALOG(I)
       GO TO 7313
 7311  CONTINUE
       S=STAN(I)*D1+AVER(I)
 7313  CONTINUE
       IF(S.LT.0.0) S=0.
        NFLW(I,JO)=S
 7314  CONTINUE
 7310  CONTINUE
       DO 7316 J=1,NDATA
       S=0.
       DO 7318 I=1,NLAST
 7318  S=S+NFLW(I,J)
       NFLW(NMON,J)=S
 7316  CONTINUE
       DO 7317 I=1,NMON
       DO 7317 J=1,NDATA
       S=NFLW(I,J)
       FLOW(I,J)=S
 7317  CONTINUE
C**
C**
C**     COMPUTE THE FIRST THREE MOMENTS
       CALL MOMENT(NMON,NDATA,NFLW,AVER,STAN,SKEW,NFLAG)
       DO 9971 I=1,NMON
       IF(SKEW(I).LE.0.0) GO TO 9972
       LU(I)=1
       GO TO 9971
 9972  LU(I)=0
 9971  CONTINUE
       ANNAV=AVER(NMON)
C**     COMPUTE A CORRELATION MATRIX
       CALL SECOR(NMON,NDATA,NFLW,COVAR)
```

```
      IF(NFLAG.NE.1) GO TO 7380
      WRITE(6,6195)
      DO 7320 I=1,NMON
 7320 WRITE(6,205) (COVAR(I,J),J=1,I)
 7380 CONTINUE
      IF(NLOG.EQ.0) GO TO 7110
C**    CHECK THE STATISTICS OF 3PLN DISTRIBUTED FLOWS
      KB=5
      CALL TRANS(NMON,NDATA,AVER,STAN,SKEW,NFLW,AVLOG,STLOG,AALOG,KB)
      CALL SECOR(NMON,NDATA,NFLW,CORRE)
      DO 3050 J=1,NLAST
      K=J+1
      JX=LU(J)
      DO 3050 I=K,NMON
      IX=LU(I)
      S=CORRE(I,J)
      KK=JX+IX
      IF(KK.EQ.1) GO TO 3052
      CORRE(I,J)=S
      GO TO 3050
 3052 CONTINUE
      S=-S
      CORRE(I,J)=S
 3050 CONTINUE
      IF(NFLAG.NE.1) GO TO 3030
      WRITE(6,6195)
      DO 3032 I=1,NMON
 3032 WRITE(6,205) (CORRE(I,J),J=1,I)
 3030 CONTINUE
 7110 CONTINUE
C**  COMPUTE PROBABILITY LEVEL OF DISTRIBUTION FUNCTION FOR EACH MONTH
      DO 211 I=1,NPRO
      D=CUMPRO(I)
      CALL PROLEL(D,STNRM)
      CDFX(I)=STNRM
  211 CONTINUE
      DO 212 I=1,NPRO
  212 PERCET(I)=100.*CUMPRO(I)
      IF(NFLAG.NE.1) GO TO 291
      WRITE(6,252) (PERCET(I),I=1,NPRO)
      WRITE(6,6248) (CDFX(I),I=1,NPRO)
      WRITE(6,6240)
      WRITE(6,252) (PERCET(I),I=1,NPRO)
  291 CONTINUE
      DO 7685 I=1,NMON
      IF(NLOG.NE.1) GO TO 7113
      JP=LU(I)
      NREV=NPRO+1
 7113 CONTINUE
      DO 213 JK=1,NPRO
      A=CDFX(JK)
      IF(NLOG.EQ.0) GO TO 7112
      S=STLOG(I)*A+AVLOG(I)
      IF(JP.EQ.0) GO TO 7115
      S=EXP(S)+AALOG(I)
      IF(S.LT.0.0) S=0.
      PFLOW(I,JK)=S
```

```
      GO TO 213
 7115 S=AALOG(I)-EXP(S)
      NJ=NREV-JK
      IF(S.LT.0.0) S=0.
      PFLOW(I,NJ)=S
      GO TO 213
 7112 CONTINUE
      S=STAN(I)*A+AVER(I)
      IF(S.LE.0.0) S=0.0
      PFLOW(I,JK)=S
  213 CONTINUE
      IF(NFLAG.NE.1) GO TO 292
      WRITE(6,253) XNAME(I,1),(PFLOW(I,J),J=1,NPRO)
  292 CONTINUE
 7685 CONTINUE
      DO 7252 I=1,NLAST
      DO 7252 J=1,I
      BBT(I,J)=STAN(I)*STAN(J)*(COVAR(I,J)-COVAR(NMON,I)*COVAR(NMON,J))
      IF(I.NE.J) GO TO 7252
      ORST(I)=SQRT(BBT(I,I))
      ACOE(I)=COVAR(NMON,I)
      S=SQRT(1.-ACOE(I)**2)
      GSKW(I)=(SKEW(I)-ACOE(I)**3*SKEW(NMON))/(S**3)
 7252 CONTINUE
C**    CONDITIONAL CORRELATION MATRIX IN THE ORIGINAL DOMAIN
      DO 223 I=1,NLAST
      DO 223 J=1,I
      A4(I,J)=BBT(I,J)/(ORST(I)*ORST(J))
  223 CONTINUE
      WRITE(6,6210)
      DO 225 I=1,NLAST
      WRITE(6,205) (A4(I,J),J=1,I)
  225 CONTINUE
      IF(NLOG.EQ.0) GO TO 3701
      DO 3002 I=1,NLAST
      SKEW(I)=GSKW(I)
 3002 CONTINUE
C**    COMPUTE CONDITIONAL COVARIANCE IN THE NORMAL DOMAIN
      DO 3036 I=1,NLAST
      S=STLOG(I)
      DO 3036 J=1,I
      S1=STLOG(J)
      BBT(I,J)=S*S1*(CORRE(I,J)-CORRE(NMON,I)*CORRE(NMON,J))
      IF(I.NE.J) GO TO 3036
      COML(I)=SQRT(BBT(I,I))
 3036 CONTINUE
C**    CONDITIONAL CORRELATION MATRIX IN THE NORMAL DOMAIN
      DO 3038 I=1,NLAST
      DO 3038 J=1,I
      A3(I,J)=BBT(I,J)/(COML(I)*COML(J))
 3038 CONTINUE
      IF(NFLAG.NE.1) GO TO 3037
      WRITE(6,6210)
      DO 3040 I=1,NLAST
 3040 WRITE(6,205) (A3(I,J),J=1,I)
 3037 CONTINUE
 3701 CONTINUE
```

```
C******
C**
      CALL CLOCK(MI,MJ,MK)
      L=MI+100*MJ+10000*MK
      L=L-L/100000*100000
      XN=2*L+1
      TEMP=RANF(XN)
C**   START FORECAST OF FLOWS BY DISAGGREGATION MODEL
C**
      IF(NFLAG.NE.1) GO TO 302
      WRITE(6,6750) (TITLE(I),I=1,20)
      WRITE(6,6193) NLOG
  302 CONTINUE
C**   SET INITIAL VALUE OF AN AGGREGATE FLOW
      DO 9989 LL=1,NNIN
      S=YINI(LL)
      A=S
      B=A
      XINI=100.*S
      CALL PROLFL(B,STNRM)
      D2=STAN(NMON)*STNRM+AVER(NMON)
      S=D2
      IF(S.LE.0.0) S=0.
      PROFW=S
      A=100.*A
      WRITE(6,260) A,PROFW
      S1=0.
      DO 220 I=1,NLAST
      D=ACOE(I)*STAN(I)/STAN(NMON)
      D=AVER(I)+D*(D2-AVER(NMON))
      ORAV(I)=D
      S1=S1+ORAV(I)
      D1=1.-ACOE(I)**2
      D1=SQRT(D1)
      ORST(I)=STAN(I)*D1
  220 CONTINUE
      WRITE(6,6214)
      DO 7752 I=1,NLAST
 7752 WRITE(6,9981) XNAME(I,1),ORAV(I),ORST(I),SKEW(I)
      WRITE(6,9981) XNAME(NMON,1),S1
      IF(NCARD.NE.1) GO TO 8117
      D=XINI
      DO 8107 I=1,NLAST
8107    WRITE(7,3) NLOG,D,XNAME(I,1),ORAV(I),ORST(I),SKEW(I)
      WRITE(7,3) NLOG,D,XNAME(NMON,1),S1
 8117 CONTINUE
      IF(NLOG.EQ.0) GO TO 7180
      DO 7132 I=1,NLAST
      D=ORAV(I)
      D1=ORST(I)
      D2=SKEW(I)
      IF(D2.LE.0.0) GO TO 7133
      LU(I)=1
      CALL LOGNOR(D,D1,D2,AV1,ST1,THIRD)
      ORAV(I)=AV1
      ORST(I)=ST1
      AALOG(I)=THIRD
```

```
      GO TO 7132
 7133 CONTINUE
      D2=-D2
      IF(D2.LT.0.005) D2=0.005
      LU(I)=0
      CALL LOGNOR(D,D1,D2,AV1,ST1,THIRD)
      S=EXP(AV1+0.5*ST1**2)
      THIRD=D+S
      ORAV(I)=AV1
      ORST(I)=ST1
      AALOG(I)=THIRD
 7132 CONTINUE
      IF(NFLAG.NE.1) GO TO 7218
      WRITE(6,310)
      DO 511 I=1,NLAST
  511 WRITE(6,255) XNAME(I,1),ORAV(I),ORST(I),AALOG(I)
 7218 CONTINUE
 7180 CONTINUE
C****
C*****
      IF(LL.GT.1) GO TO 9997
      IF(NLOG.EQ.0) GO TO 3705
C**    THEORETICAL CONDITIONAL COVARIANCE MATRIX IN THE LOG DOMAIN
      DO 7134 I=1,NLAST
      S=EXP(ORST(I)**2)-1.
      S=SQRT(S)
      DO 7134 J=1,I
      S1=EXP(ORST(J)**2)-1.
      S1=SQRT(S1)
      D=1.+A4(I,J)*S*S1
      DIFF(I,J)=ALOG(D)
      DIFF(J,I)=DIFF(I,J)
 7134 CONTINUE
      DO 3054 I=1,NLAST
      DO 3054 J=1,I
      A4(I,J)=DIFF(I,J)/(ORST(I)*ORST(J))
 3054 CONTINUE
      IF(NFLAG.NE.1) GO TO 75
      WRITE(6,6263)
      DO 3044 I=1,NLAST
 3044 WRITE(6,206) (DIFF(I,J),J=1,I)
      WRITE(6,6263)
      DO 3056 I=1,NLAST
 3056 WRITE(6,205) (A4(I,J),J=1,I)
   75 CONTINUE
      DO 3042 I=1,NLAST
      DO 3042 J=1,I
      BBT(I,J)=A3(I,J)*ORST(I)*ORST(J)
 3042 CONTINUE
 3705 CONTINUE
      DO 3007 I=1,NLAST
      DO 3007 J=1,I
      DIFF(I,J)=BBT(I,J)
 3007 CONTINUE
      IF(NFLAG.NE.1) GO TO 7214
      WRITE(6,6210)
      DO 7254 I=1,NLAST
```

```
 7254 WRITE(6,206) (BBT(I,J),J=1,I)
 7214 CONTINUE
      IF(NLOG.EQ.1) GO TO 3046
      NCH=NLAST
      NLAST=NSON
 3046 CONTINUE
      KOT=0
      DO 70 I=1,NLAST
      DO 70 J=1,I
      KOT=KOT+1
      AAB(KOT)=BBT(I,J)
   70 CONTINUE
C**
C**   SYMMETRIC MATRIX (= BBT ) IS DIAGONALIZED BY THE EIGENVALUES AND
C**   EIGENVECTORS (=BCOE)
      NQ=12
      IJOB=2
      CALL EIGRS(AAB,NLAST,IJOB,ROOT,Z,NQ,WK,IER)
      WRITE(6,201) IER
      WRITE(6,203) (ROOT(I),I=1,NLAST)
      DO 7730 I=1,NLAST
      S=ROOT(I)
      IF(S.LT.0.0) GO TO 9999
      EIGEN(I)=SQRT(S)
 7730 CONTINUE
      DO 7731 I=1,NLAST
      DO 7731 J=1,NLAST
      BCOE(I,J)=Z(I,J)
      BINV(J,I)=BCOE(I,J)
 7731 CONTINUE
C**    COMPUTE THE DETERMINANT OF AN EIGENVECTOR MATRIX
      CALL DECOM(Z,NQ,NLAST,AAB,WK,DT1,VIPDA)
      DET=DT1
      DO 72 I=1,NLAST
   72 DET=DET*Z(I,I)
      WRITE(6,208) DET
      IF(DET.GT.0.0) GO TO 74
      DO 7736 I=1,NLAST
 7736 BCOE(I,1)=-BCOE(I,1)
      DO 7732 I=1,NLAST
      DO 7732 J=1,NLAST
      BINV(J,I)=BCOE(I,J)
 7732 CONTINUE
   74 CONTINUE
      IF(NLOG.EQ.1) GO TO 3048
      NLAST=NCH
      DO 6020 I=1,NLAST
      DO 6020 J=1,I
 6020 BBT(I,J)=DIFF(I,J)
      D1=BBT(NLAST,NLAST)
      DO 6000 I=1,NSON
      S=0.
      DO 6001 K=1,NSON
 6001 S=S+BINV(I,K)*BBT(NLAST,K)
      S=S/(EIGEN(I)*EIGEN(I))
      BCOE(NLAST,I)=S
      BCOE(I,NLAST)=0.
```

```
      D1=D1-(BCOE(NLAST,I)*EIGEN(I))**2
 6000 CONTINUE
      IF(D1.LT.0.0) D1=1.0E-16
      D1=SQRT(D1)
      EIGEN(NLAST)=D1
      D1=1.
      BCOE(NLAST,NLAST)=D1
      D1=1./D1
      BINV(NLAST,NLAST)=D1
      DO 4340 I=1,NSON
      S=0.
      DO 4341 K=1,NSON
 4341 S=S+BCOE(NLAST,K)*BINV(K,I)
      S=-D1*S
      BINV(NLAST,I)=S
      BINV(I,NLAST)=0.
 4340 CONTINUE
 3048 CONTINUE
      IF(NFLAG.NE.1) GO TO 8112
C**    CHECK OF INVERSE MATRIX OF ROTATION MATRIX
      KCH=2
      CALL CHECK(NLAST,BINV,BCOE,KCH)
 8112 CONTINUE
C**    CHECK THE VARIANCES USING ROTATION MATRIX AND EIGENVALUES
      DO 7256 I=1,NLAST
      DO 7256 J=1,I
      S=0.
      DO 7257 K=1,NLAST
 7257 S=S+(BCOE(I,K)*BCOE(J,K))*EIGEN(K)**2
      A3(I,J)=S
      IF(I.NE.J) GO TO 7256
      Y(I)=S
      GSTN(I)=SQRT(S)
 7256 CONTINUE
      STLAST=Y(NLAST)
      IF(NFLAG.NE.1) GO TO 7814
      WRITE(6,4362)
      WRITE(6,206) (EIGEN(I),I=1,NLAST)
      WRITE(6,6254)
      DO 4343 I=1,NLAST
 4343 WRITE(6,206) (A3(I,J),J=1,I)
      WRITE(6,6243) (GSTN(J),J=1,NLAST)
 7814 CONTINUE
      IF(NFLAG.NE.1) GO TO 7216
      WRITE(6,209)
      DO 7734 I=1,NLAST
 7734 WRITE(6,206) (BCOE(I,J),J=1,NLAST)
 7216 CONTINUE
 9997 CONTINUE
C****
C****
C**    CONDITIONAL MARGINAL DISTRIBUTION (** CONDITIONAL MEAN AND
C**    VARIANCE **)
      IF(NFLAG.NE.1) GO TO 318
      WRITE(6,6245)
      WRITE(6,252) (PERCET(KK),KK=1,NPRO)
  318 CONTINUE
```

```
      DO 321 I=1,NLAST
      IF(NLOG.NE.1) GO TO 3025
      JP=LU(I)
      NREV=NPRO+1
 3025 CONTINUE
      PREAV=ORAV(I)
      PREST=ORST(I)
      DO 323 JK=1,NPRO
      STNRM=CDFX(JK)
      S=PREAV+STNRM*PREST
      IF(NLOG.EQ.0) GO TO 320
      IF(JP.EQ.0) GO TO 3017
      S=EXP(S)+AALOG(I)
      IF(S.LT.0.0) S=0.
      Y(JK)=S
      GO TO 323
 3017 CONTINUE
      NJ=NREV-JK
      S=AALOG(I)-EXP(S)
      IF(S.LT.0.0) S=0.
      Y(NJ)=S
      GO TO 323
  320 CONTINUE
      IF(S.LE.0.0) S=0.
      Y(JK)=S
  323 CONTINUE
      IF(NFLAG.NE.1) GO TO 324
      WRITE(6,253) XNAME(I,1),(Y(K),K=1,NPRO)
  324 CONTINUE
      IF(NCARD.NE.1) GO TO 321
      D=XINI
        WRITE(7,2) NLOG,D,XNAME(I,1),(Y(K),K=1,NHAF)
        WRITE(7,2) NLOG,D,XNAME(I,1),(Y(K),K=NHAF1,NPRO)
  321 CONTINUE
C***********
C**    CONDITIONAL DISTRIBUTION (** CONDITIONAL MEAN AND
C**    VARIANCE-COVARIANCE MATRIX **)
C**    RESULTS OF ORTHOGONAL TRANSFORMATION BY PRINCIPAL COMPONENTS METH
      IF(NFLAG.NE.1) GO TO 295
      WRITE(6,6246)
      WRITE(6,252) (PERCET(KK),KK=1,NPRO)
  295 CONTINUE
C**   COMPUTE PROBABILITY LEVEL OF FORECAST FLOW
      DO 227 I=1,NLAST
      PREST = EIGEN(I)
      DO 228 JK=1,NPRO
      STNRM=CDFX(JK)
      S=PREST*STNRM
      A4(I,JK)=S
  228 CONTINUE
  227 CONTINUE
      DO 7755 I=1,NLAST
      IF(NLOG.NE.1) GO TO 3029
      JP=LU(I)
 3029 CONTINUE
      DO 7757 J=1,NPRO
      S=0.
```

```
      DO 7756 K=1,NLAST
7756  S=S+BCOE(I,K)*A4(K,J)
      S=S+DRAV(I)
      IF(NLOG.EQ.0) GO TO 7124
      IF(JP.EQ.0) GO TO 3019
      S=EXP(S)+AALOG(I)
      GO TO 7124
3019  S=AALOG(I)-EXP(S)
7124  CONTINUE
      IF(S.LE.0.0) S=0.
      Y(J)=S
7757  CONTINUF
      B=Y(1)
      C=Y(NPRO)
      IF(B.LT.C) GO TO 298
      NREV=NPRO+1
      DO 7128 J=1,NPRO
      NJ=NREV-J
      PDIS(I,NJ)=Y(J)
7128  CONTINUE
      GO TO 73
 298  CONTINUE
      DO 7130 J=1,NPRO
      PDIS(I,J)=Y(J)
7130  CONTINUE
  73  CONTINUE
      IF(NFLAG.NE.1) GO TO 296
      WRITE(6,253) XNAME(I,1),(PDIS(I,KL),KL=1,NPRO)
 296  CONTINUE
7755  CONTINUE
      IF(NCARD.NE.1) GO TO 8115
      D=XINI
      DO 8105 I=1,NLAST
8105    WRITE(7,2) NLOG,D,XNAME(I,1),(PDIS(I,KL),KL=1,NHAF)
      DO 8106 I=1,NLAST
8106    WRITE(7,2) NLOG,D,XNAME(I,1),(PDIS(I,KL).KL=NHAF1,NPRO)
8115  CONTINUE
C******
C******
       IF(NSIM.EQ.0) GO TO 10000
C**    SET OUT MONTE CARLO EXPERIMENTS WHEN CONDITIONAL MEAN AND
C**    VARIANCE-COVARIANCE MATRICES ARE GIVEN
      ND10=290
      WRITE(6,6250) (TITLE(I),I=1,20)
      WRITE(6,6193) NLOG
      WRITE(6,6259) NDATB
6259  FORMAT(1H0,10X,*$$$+++ MONTE CARLO SIMULATION OF CONDITIONAL PROBA
     1BILITY FLOW +++$$$*/1H0,10X,*NUMBER OF GENERATED FLOWS =*,I5)
      IF(NCARD.NE.1) GO TO 346
      WRITE(7,640) NLOG,ND10,XINI
 640  FORMAT(8X,*LOG =*,I3,*  DATA =*,I5,*  INI =*,F8.0)
 346  CONTINUE
C**    START NORMAL AND 3PLN GENERATORS
      DO 8710 JJ=1,NDATB
      DO 8712 I=1,NLAST
      CALL NORDEVI(STNRM)
      WK(I)=STNRM*EIGEN(I)
```

```
 8712 CONTINUE
      DO 8714 I=1,NLAST
      IF(NLOG.NE.1) GO TO 3709
      JP=LU(I)
 3709 CONTINUE
      S=0.
      DO 8715 K=1,NLAST
 8715 S=S+BCOE(I,K)*WK(K)
      IF(NLOG.NE.1) GO TO 3711
      S=ORAV(I)+S
      IF(JP.EQ.0) GO TO 9991
      S=EXP(S)+AALOG(I)
      GO TO 81
 9991 CONTINUE
      S=AALOG(I)-EXP(S)
      GO TO 81
 3711 S=ORAV(I)+S
   81 CONTINUE
      IF(S.LT.0.0) S=0.
      SEFLOW(I,JJ)=S
 8714 CONTINUE
 8710 CONTINUE
       IF(NLOG.EQ.0) GO TO 8754
      DO 8748 I=1,NTEST
      LU(I)=1
 8748 CONTINUE
 8754   CONTINUE
C**    COMPUTE THE FIRST THREE CONDITIONAL MOMENTS
      D=XINI
      S1=0.
      DO 8739 I=1,NTEST
      DO 8740 J=1,NDATB
 8740 Y(J)=SEFLOW(I,J)
      CALL QKRSRT(Y,NDATB)
      CALL MOMENT1(NDATB,Y,AVZ,STZ,SKZ)
      IF(NLOG.EQ.0) GO TO 8756
      IF(SKZ.LT.0.0) LU(I)=0
8756    CONTINUE
      GAVE(I)=AVZ
      GSTN(I)=STZ
      GSKW(I)=SKZ
      S1=S1+AVZ
C**    PICK UP PERCENTILE VALUES OF CONDITIONAL MARGINAL DISTRIBUTION
C**    CORRESPONDING TO CUMULATIVE PROBABILITY LEVELS
      DO 8724 IJ=1,NPRO
      NR=CUMPRO(IJ)*NDATB
      WK(IJ)=Y(NR)
 8724 CONTINUE
      WRITE(6,252) (PERCET(KK),KK=1,NPRO)
      WRITE(6,253) XNAME(I,1),(WK(KK),KK=1,NPRO)
      IF(NCARD.NE.1) GO TO 8716
      WRITE(7,2) NLOG,D,XNAME(I,1),(WK(KK),KK=1,NHAF)
      WRITE(7,2) NLOG,D,XNAME(I,1),(WK(KK),KK=NHAF1,NPRO)
 8716 CONTINUE
      WRITE(6,215)
C**    OF 2000 GENERATED SEASONAL FLOWS, 290 DATA (120 IN THE LOWER TAIL
C**    AND 50 IN THE MIDDLE AND 120 IN THE UPPER TAIL) ARE SELECTED
```

```
      DO 641 J=1,ND10
      IF(J.GT.120) GO TO 643
      IR=1+5*(J-1)
      S=Y(IR)
      Y(J)=S
      GO TO 641
  643 IF(J.GT.170) GO TO 645
      IR=600+16*(J-120)
      S=Y(IR)
      Y(J)=S
      GO TO 641
  645 IR=1400+5*(J-170)
      S=Y(IR)
      Y(J)=S
  641 CONTINUE
      WRITE(6,632) (Y(J),J=1,ND10)
  632 FORMAT(1H ,10X,10F10.0)
      IF(NCARD.NE.1) GO TO 8723
      WRITE(7,634) (Y(J),J=1,ND10)
  634 FORMAT(10F8.0)
 8723 CONTINUE
 8739 CONTINUE
      WRITE(6,6214)
      DO 509 I=1,NLAST
  509 WRITE(6,9981) XNAME(I,1),GAVE(I),GSTN(I),GSKW(I)
      WRITE(6,9981) XNAME(NMON,1),S1
      IF(NCARD.NE.1) GO TO 8713
      DO 513 I=1,NTEST
  513     WRITE(7,3) NLOG,D,XNAME(I,1),GAVE(I),GSTN(I),GSKW(I)
          WRITE(7,3) NLOG,D,XNAME(NMON,1),S1
 8713 CONTINUE
C**    COMPUTE CONDITIONAL CORRELATION MATRIX OF GENERATED DATA
      XDATB=NDATB
      WRITE(6,6210)
      DO 8742 K=1,NTEST
      DO 8742 J=1,K
      S=0.
      DO 8743 L=1,NDATB
 8743 S=S+(SEFLOW(K,L)-GAVE(K))*(SEFLOW(J,L)-GAVE(J))
      S=S/(XDATB-1.)
      S1=S/(GSTN(K)*GSTN(J))
      A4(K,J)=S1
 8742 CONTINUE
      IF(NLOG.EQ.0) GO TO 8758
      DO 8750 J=1,NSON
      K=J+1
      JX=LU(J)
      DO 8750 I=K,NLAST
      IX=LU(I)
      S=A4(I,J)
      KK=JX+IX
      IF(KK.EQ.1) GO TO 8752
      A4(I,J)=S
      GO TO 8750
 8752 CONTINUE
      S=-S
      A4(I,J)=S
```

```
 8750 CONTINUE
 8758    CONTINUE
      DO 8735 K=1,NTEST
      WRITE(6,205) (A4(K,J),J=1,K)
      IF(NCARD.NE.1) GO TO 8735
      WRITE(7,4) XNAME(K,1),(A4(K,J),J=1,K)
 8735 CONTINUE
C**     FINISH NORMAL AND 3PLN GENERATORS
10000      CONTINUE
 9989 CONTINUE
C*******
C*******
2       FORMAT(I8,F4.0,4X,A4,5F10.0)
3       FORMAT(I8,F4.0,4X,A4,2F10.0,F10.3)
4       FORMAT(10X,A4,7F8.3)
  101 FORMAT(16F5.2)
  103 FORMAT(8F10.3)
 6191 FORMAT(1H ,I20,F13.3,F15.3)
 6193 FORMAT(1H0,10X,*ASSUMED DISTRIBUTION OF FLOWS AT EACH TIME PERIOD*
     1/1H0,10X,*IF(NLOG = 0) NORMAL DISTRIBUTION  IF(NLOG = 1) 3PLN DIST
     2RIBUTION*/1H0,10X,*NLOG =*,I3)
 6195 FORMAT(1H0,20X,*CORRELATION MATRIX OF INPUT DATA*/)
 6200 FORMAT(1H0,20X,*VARIANCE-COVARIANE MATRIX*/)
 6204 FORMAT(1H ,I19,E12.3,E16.3)
 6210 FORMAT(1H0,20X,*CONDITIONAL VARIANCE AND COVARIANCE MATRIX*/)
 6212 FORMAT(1H0,20X,*MEAN AND STANDARD DEVIATION OF FORECAST FLOW*/1H0,
     124X,*      MEAN     ST. DE.*/)
 6214 FORMAT(1H0,10X,*CONDITIONAL MEAN AND STANDARD DEVIATION AND SKEWNE
     1SS IN THE REAL DOMAIN*/)
  215 FORMAT(1H0)
  201 FORMAT(1H0,10X,*PERFORMANCE INDEX =*,I7)
  203 FORMAT(1H0,10X,*EIGENVALUES*/1H0,10X,12F10.3)
  205 FORMAT(1H ,F14.3,11F9.3)
  206 FORMAT(1H ,11E12.4)
  208 FORMAT(1H0,10X,*DETERMINANT OF AN EIGENVECTORS MATRIX =*,E20.5)
  209 FORMAT(1H0,10X,*ROTATION MATRIX*/)
 7700 FORMAT(16I5)
 7702 FORMAT(14X,11F6.0)
 7704 FORMAT(3A4)
 7705 FORMAT(20A4)
  252 FORMAT(1H0,8X,F3.0,*TH PER*,F5.0,*TH PER*,F5.0,*TH PER*,F5.0,*TH P
     1ER*,F5.0,*TH PER*,F5.0,*TH PER*,F5.0,*TH PER*,F5.0,*TH PER*,F5.0,*
     2TH PER*/)
  253 FORMAT(1H ,4X,A4,F9.0,9F11.0)
  254 FORMAT(1H ,20X,A4,F9.0,4F11.0)
  255 FORMAT(1H ,20X,A4,E14.4,2E15.4)
  256 FORMAT(1H ,20X,I4,E14.4,E15.4)
  260 FORMAT(1H0,10X,*INITIAL VALUE =*,F3.0,*TH PERCENTILE FLOW =*F9.0/)
 6240 FORMAT(1H0,20X,*PERCENTILE FLOWS OF UNCONDITIONAL (NO-FORECAST) DI
     1STRIBUTION FUNCTION*/)
 6243 FORMAT(1H0,20X,*REPRODUCED CONDITIONAL STANDARD DEVIATION IN THE N
     1ORMAL DOMAIN*/1H0,12E11.3)
 6245 FORMAT(1H0,10X,*CONDITIONAL MARGINAL DISTRIBUTION (INCORPORATION O
     1F CONDITIONAL MEAN AND VARIANCE)*/)
 6246 FORMAT(1H0,10X,*CONDITIONAL DISTRIBUTION (INCORPORATION OF CONDITI
     1ONAL MEAN AND VARIANCE-COVARIANCE*/)
 6248 FORMAT(1H ,8X,F9.3,10F11.3)
```

```
6250 FORMAT(1H1,10X,20A4)
6254 FORMAT(1H0,10X,*REPRODUCED CONDITIONAL VARI-COVAR MATRIX*/)
6263 FORMAT(1H0,10X,*THEORETICAL CONDITIONAL COVARIANCE MATRIX IN THE L
    1OG DOMAIN (3PLN)*/)
9981 FORMAT(1H ,10X,A4,2F10.0,F10.3)
 310 FORMAT(1H0,10X,*CONDITIONAL MEAN AND ST. DEVIATION IN LOG DOMAIN A
    1ND THIRD PARAMETER*/)
4362 FORMAT(1H0,10X,*STANDARD DEVIATION OF ORTHOGONAL COMPONENTS*/)
5102 FORMAT(1H0,10X,*MODIFIED CORRELATION COEFFICIENTS*/)
9999 CONTINUE
     STOP
     END
```

```
      SUBROUTINE MOMENT(NS2,ND11,RR,AV,SD,SK,KK)
      DIMENSION RR(13,65),AV(13),SD(13),SK(13),CV(13),SB(13)
      XD11=ND11
      DO 11 I=1,NS2
      SUM=SUM2=SUM3=0.0
      DO 10 J=1,ND11
      SUM=SUM+RR(I,J)
      SUM2=SUM2+RR(I,J)*RR(I,J)
   10 SUM3=SUM3+RR(I,J)*RR(I,J)*RR(I,J)
      AV(I)=SUM/XD11
      SUM3=SUM3/XD11
      SUM3=SUM3-3.*AV(I)*(SUM2/XD11)+2.*AV(I)**3.
      SD(I)=1./(XD11-1.)*(SUM2-XD11*AV(I)*AV(I))
      SB(I)=SD(I)*(XD11-1.)/XD11
      SD(I)=SQRT(SD(I))
      SB(I)=SQRT(SB(I))
   11 SK(I)=SUM3/(SB(I)**3.)
      IF(KK.NE.1) GO TO 31
      DO 12 I=1,NS2
   12 CV(I)=SD(I)/AV(I)
      WRITE(6,32)
      DO 29 I=1,NS2
   29 WRITE(6,30) I,AV(I),SD(I),CV(I),SK(I)
      GO TO 42
   31 IF(KK.NE.2) GO TO 42
      WRITE(6,40)
      DO 41 I=1,NS2
   41 WRITE(6,30) I,AV(I),SD(I),SK(I)
   30 FORMAT(1H ,I17,F11.2,F15.3,F17.4,F15.4)
   32 FORMAT(1H0,20X,*    MEAN   STANDARD DE.   CO. VARIATION  SKEWNESS
     1  CO.*,/)
   40 FORMAT(1H0,20X,*    MEAN   STANDARD DE.   SKEWNESS  COE.*,/)
   42 RETURN
      END
```

```
      SUBROUTINE TRANS(N,ND,AV,ST,SW,XLOW,AVY,STY,PA3,KKX)
      DIMENSION AV(13),ST(13),SW(13),XLOW(13,65),AVY(13),STY(13),PA3(13)
      DIMENSION AVW(13),STW(13),SWY(13)
      DO 7721 I=1,N
      D=AV(I)
      D1=ST(I)
      D2=SW(I)
      IF(D2.LE.0.0) GO TO 7722
      JP=1
      CALL SKEWLOG(ND,D2)
      SW(I)=D2
      CALL LOGNOR(D,D1,D2,AV1,ST1,THIRD)
      AVY(I)=AV1
      STY(I)=ST1
      PA3(I)=THIRD
      GO TO 7723
 7722 CONTINUE
      IF(KKX.NE.5) GO TO 12
      D2=-D2
      IF(D2.LT.0.005) D2=0.005
      JP=0
      CALL SKEWLOG(ND,D2)
      SW(I)=-D2
      CALL LOGNOR(D,D1,D2,AV1,ST1,THIRD)
      S=EXP(AV1+0.5*ST1**2)
      THIRD=D+S
      AVY(I)=AV1
      STY(I)=ST1
      PA3(I)=THIRD
      GO TO 7723
   12 CONTINUE
      SW(I)=D2
      AVY(I)=0.
      STY(I)=0.
      PA3(I)=0.
      GO TO 7721
 7723 CONTINUE
      DO 7724 J=1,ND
      IF(JP.EQ.0) GO TO 553
      DD=XLOW(I,J)-THIRD
      GO TO 555
  553 CONTINUE
      DD=THIRD-XLOW(I,J)
  555 CONTINUE
      IF(DD.LE.0.0) GO TO 859
      DD=ALOG(DD)
      XLOW(I,J)=DD
      GO TO 7724
  859 DD=0.
      XLOW(I,J)=DD
 7724 CONTINUE
 7721 CONTINUE
      IF(KKX.EQ.0) GO TO 33
      WRITE(6,900)
      DO 901 I=1,N
  901 WRITE(6,902) I,AVY(I),STY(I),PA3(I),SW(I)
      WRITE(6,903)
```

```
 33 CONTINUE
    CALL MOMENT(N,ND,XLOW,AVW,STW,SWY,KKX)
900 FORMAT(1HO,20X,*THEORETICAL MEAN AND STANDARD DEVIATION AND THIRD
   1PARAMETER*/,20X,*     MEAN    STANDARD DE.  THIRD PARAMETER   CORREC
   2TED SKEW*/)
902 FORMAT(1H ,I15,4E15.6)
903 FORMAT(1HO,20X,*THE FIRST THREE MOMENTS OF LOG(X - A)*)
    RETURN
    END
```

```
     SUBROUTINE CORMAT(N1,N,FLOW,COR)
     DIMENSION FLOW(2,65),AV(2),ST(2)
     XN=N
     DO 10 I=1,N1
     SUM=SUM2=0.0
     DO 11 J=1,N
     SUM=SUM+FLOW(I,J)
  11 SUM2=SUM2+FLOW(I,J)*FLOW(I,J)
     SUM=SUM/XN
     SUM2=1./(XN-1.)*(SUM2-XN*SUM*SUM)
     SUM2=SQRT(SUM2)
     AV(I)=SUM
  10 ST(I)=SUM2
     COR=0.0
     DO 12 J=1,N
  12 COR=COR+(FLOW(2,J)-AV(2))*(FLOW(1,J)-AV(1))
     COR=COR/(XN-1.)
     COR=COR/(ST(1)*ST(2))
     RETURN
     END
```

```
     SUBROUTINE SKEWLOG(N,BIAS)
     XN=N
     A=1.01+7.01/XN+14.66/(XN*XN)
     B=1.69/XN+74.66/(XN*XN)
     BIAS=BIAS*(A+B*BIAS**3.)
     RETURN
     END
```

```
      SUBROUTINE SECOR(NL,ND,SFLOW,CORR)
      DIMENSION SFLOW(13,65),CORR(12,12),BLG(2,65)
      N1=NL-1
      DO 4114 I=1,NL
 4114 CORR(I,I)=1.0
      DO 4115 J=1,N1
      JJ=J+1
      DO 4116 K=1,ND
 4116 BLG(2,K)=SFLOW(J,K)
      DO 4117 I=JJ,NL
      DO 4118 K=1,ND
 4118 BLG(1,K)=SFLOW(I,K)
      NM=2
      CALL CORMAT(NM,ND,BLG,A)
      CORR(I,J)=A
      CORR(J,I)=CORR(I,J)
 4117 CONTINUE
 4115 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LOGNOR(AV,ST,SK,AVY,STY,ACON)
C**   COMPUTE MEAN AND STANDARD DEVIATION IN THE LOG DOMAIN AND
C**   THIRD PARAMETER BY THE 3PLN DISTRIBUTION
      A=1.+SK*SK/2.
      AB=SQRT(A**2-1.)
      AC=A+AB
      AD=A-AB
      APLUS=AC**0.3333333
      AMINUS=AD**0.3333333
      YEXP=APLUS+AMINUS-1.
      VAY=ALOG(YEXP)
      STY=SQRT(VAY)
      YEXP=YEXP-1.
      F=ST*ST/YEXP
      F=ALOG(F)
      AVY=0.5*(-VAY+F)
      ACON=0.5*VAY+AVY
      ACON=EXP(ACON)
      ACON=AV-ACON
      RETURN
      END
```

```
      SUBROUTINE QKRSRT(NA,JJ)
      DIMENSION NA(JJ),NLT(120),NUT(120)
      LOGICAL LE2,GE2
      REAL NA,NT,NX
*
*     QUICKERSORT IS A WAY OF SORTING ELEMENTS IN AN ARRAY INTO
*     ASCENDING ORDER. THE ARRAY IS CONTINUALLY SPLIT INTO
*     PARTS SUCH THAT THE ELEMENTS OF ONE PART ARE LESS THAN
*     ALL ELEMENTS OF THE OTHER, WITH A THIRD PART IN THE
*     MIDDLE CONSISTING OF A SINGLE ELEMENT. THIS METHOD IS
*     CONSIDERABLY FASTER THAN THE STANDARD COMPARISON-TRANS-
*     POSITION METHOD WHERE THE NUMBER OF COMPARISONS IS ON
*     THE ORDER OF N≥%ARRAY SIZE↑ SQUARED. QUICKERSORT REQUIRES
*     ONLY ≥ON THE ORDER OF↑ N * LOG N COMPARISONS.
*     THE CALL IS :      CALL QKRSRT≥NA,N↑
*                  WHERE NA% ARRAY NAME
*                         N% ARRAY LENGTH.
*     IN THIS VERSION THERE IS ESSENTIALLY NO LIMIT TO THE
*     LENGTH OF THE SORTED ARRAY. ≥THE ABSOLUTE LIMIT IS E TO
*     THE 13.8+ I.E. LARGER THAN ANY INTERNALLY STORED ARRAY
*     IN THE GE 635.↑ ANY SORT OF ALPHANUMERIC DATA IN THE
*     ARRAY WILL BE SORTED.
*
*     QUICKERSORT IS ALGORITHM = 271, COLLECTED ALGORITHMS
*     FROM CACM.
*
      J=JJ
      I=1
      M=1
   10 I1=I+1
      IF(J.LE.I1) GO TO 90
      NP=(J+I)/2
      NT=NA(NP)
      NA(NP)=NA(I)
      NQ = J
      K=I
   15 K=K+1
      IF(K.GT.NQ) GO TO 50
      IF(NA(K).LE.NT) GO TO 15
      NQ=NQ+1
   20 NQ=NQ-1
      IF(NQ.LT.K) GO TO 30
      IF(NA(NQ).GE.NT) GO TO 20
      NX=NA(K)
      NA(K) = NA(NQ)
      NA(NQ)=NX
      NQ=NQ-1
      GO TO 15
   30 NQ=K-1
   50 NA(I)=NA(NQ)
      NA(NQ)=NT
      IF(2*NQ-I-J) 70,70,60
   60 NLT(M)=I
      NUT(M)=NQ-1
      I=NQ+1
      GO TO 80
   70 NLT(M)=NQ+1
```

```
      NUT(M)=J
      J=NQ-1
   80 M=M+1
      GO TO 10
   90 IF(I.GE.J) GO TO 100
      IF(NA(I).LE.NA(J)) GO TO 100
      NX=NA(I)
      NA(I)=NA(J)
      NA(J)=NX
  100 M=M-1
      IF(M.EQ.0) RETURN
      I=NLT(M)
      J=NUT(M)
      GO TO 10
      END
```

```
      SUBROUTINE CHECK(N,YCHK,XCHK,KK)
C     CHECK PROGRAM OF TRANSPOSE OR INVERSE MATRIX OF B
      DIMENSION YCHK(12,12),XCHK(12,12),DZ(12,12)
      DO 171 I=1,N
      DO 171 J=1,I
      S=0.0
      DO 172 K=1,N
  172 S=S+YCHK(I,K)*XCHK(K,J)
  171 DZ(I,J)=S
      IF(KK.NE.1) GO TO 175
      WRITE(6,350)
      DO 173 I=1,N
  173 WRITE(6,335) (DZ(I,J),J=1,I)
  335 FORMAT(1H ,12E11.3)
      GO TO 179
  175 WRITE(6,351)
      DO 176 I=1,N
  176 WRITE(6,332) (DZ(I,J),J=1,I)
  332 FORMAT(1H ,F14.4,11F9.4)
  350 FORMAT(1HO,20X,*TEST OF BBT*/)
  351 FORMAT(1HO,20X,*TEST OF INVERSE MATRIX OF B*,/)
  179 RETURN
      END
```

```
      SUBROUTINE NORDEVI(ANORM)
C**   GENERATE STANDARD NORMAL DEVIATE (N(0,1)) AFTER HASTINGS
      START=0.0
      TEMP=RANF(START)
      FAC=-1.
      IF(TEMP.LT.0.5) GO TO 70
      FAC=1.
      TEMP=1.-TEMP
   70 ARG=1./TEMP/TEMP
      ARG=ALOG(ARG)
      TX=SQRT(ARG)
      ANUM=2.515517+TX*(0.802853+0.010328*TX)
      DEN=1.+1.432788*TX+TX*TX*(0.189269+0.001308*TX)
      ANORM=(TX-ANUM/DEN)*FAC
      RETURN
      END
```

```
      SUBROUTINE MOMENT1(ND11,RR,AV,SD,SK)
C**   COMPUTE THE FIRST THREE MOMENTS OF FLOWS
      DIMENSION RR(ND11)
      XD11=ND11
      SUM=SUM2=SUM3=0.
      DO 10 J=1,ND11
      SUM=SUM+RR(J)
      SUM2=SUM2+RR(J)*RR(J)
   10 SUM3=SUM3+RR(J)*RR(J)*RR(J)
      AV=SUM/XD11
      SUM3=SUM3/XD11
      SUM3=SUM3-3.*AV*(SUM2/XD11)+2.*AV**3.
      SD=1./(XD11-1.)*(SUM2-XD11*AV*AV)
      SB=SD*(XD11-1.)/XD11
      SD=SQRT(SD)
      SB=SQRT(SB)
      SK=SUM3/(SB**3.)
      RETURN
      END
```

```
      SUBROUTINE ORDER(N,X,NODER)
C**   DESCENDING ORDER OBSERVATIONS
      DIMENSION X(N),NODER(N)
      DO 650 I=1,N
  650 NODER(I)=I
      J=1
  750 I=J+1
      XU=X(J)
  730 IF(X(I).LE.XU) GO TO 710
      XU=X(I)
      K=I
      KI=NODER(I)
  710 I=I+1
      IF(I.LE.N) GO TO 730
      IF(X(J).EQ.XU) GO TO 740
      XY=X(J)
      IX=NODER(J)
      X(J)=XU
      NODER(J)=KI
      X(K)=XY
      NODER(K)=IX
  740 J=J+1
      IF(J.LT.N) GO TO 750
      RETURN
      END
```

```
      SUBROUTINE PROLEL(TEMP,ANORM)
      FAC=-1.
      IF(TEMP.LT.0.5) GO TO 70
      FAC=1.
      TEMP=1.-TEMP
   70 ARG=1./TEMP/TEMP
      ARG=ALOG(ARG)
      TX=SQRT(ARG)
      ANUM=2.30753+0.27061*TX
      DEN=1.+TX*(0.99229+0.04481*TX)
      ANORM=(TX-ANUM/DEN)*FAC
      RETURN
      END
```